# BDA016 Stavební mechanika 2
# 11. přednáška

- Zjednodušená deformační metoda – rovinný rám

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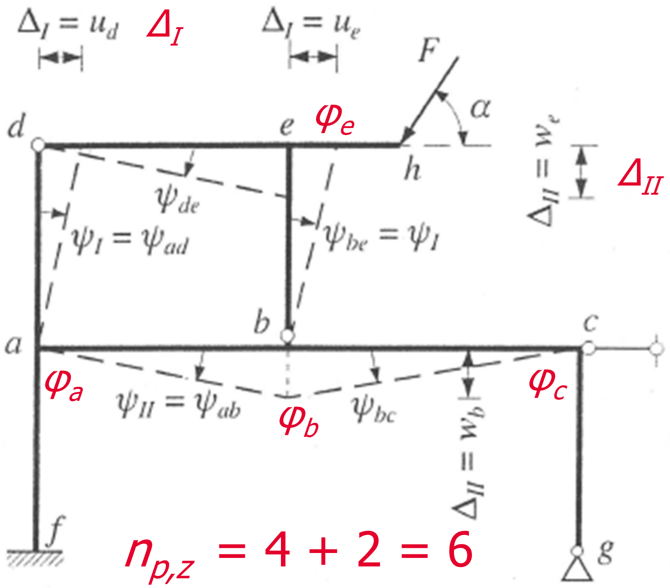
V přednášce jsou použity obrázky z učebnice Kadlčák, J., Kytýr, J. Statika stavebních konstrukcí II. Staticky neurčité prutové konstrukce. Nakladatelství VUTIUM v Brně, 2004.

# ZJEDNODUŠENÁ DEFORMAČNÍ METODA




- pro každý monolitický styčník se sestaví **momentová podmínka** rovnováhy

  $M_{ab} + M_{ad} + M_{af} = 0;\ M_{ba} + M_{bc} = 0$

  $M_{cb} + M_{cg} = 0;\ M_{ed} + M_{ed} + M_{eh} = 0$

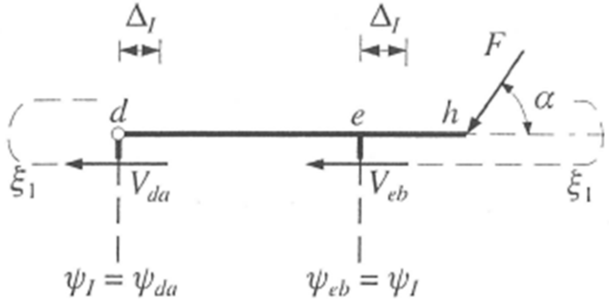
- koncové momenty $M$ se vyjádří pomocí $\varphi, \Psi$ $\rightarrow$ **styčníkové rovnice**
- pro posuny uvolněných částí soustavy se sestaví **součtové podmínky** rovnováhy

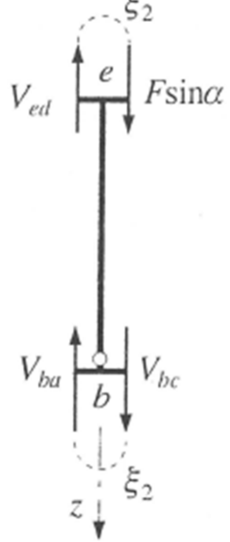
  $\sum F_x = 0: -V_{da} - V_{eb} - F \cdot \cos\alpha = 0$

  $\sum F_z = 0:$

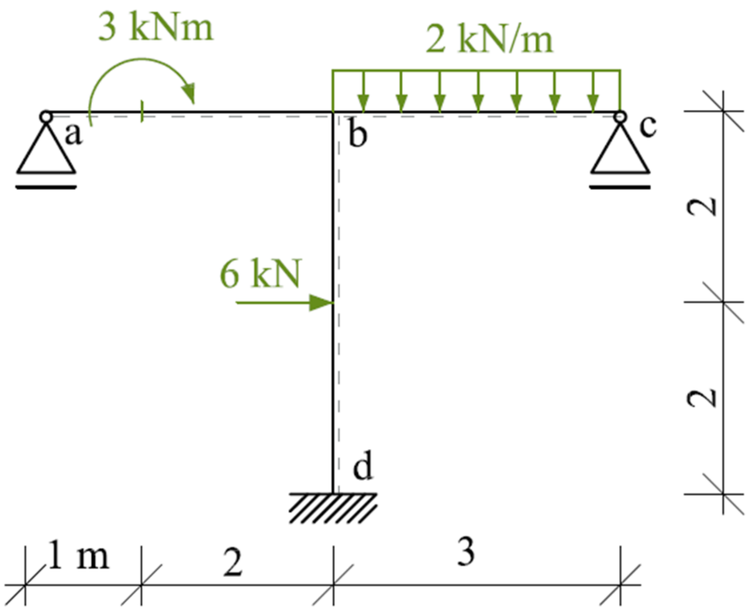
  $-V_{ba} + V_{bc} - V_{ed} + F \cdot \sin\alpha = 0$

- posouvající síly $V$ se vyjádří pomocí $M\ (\varphi, \Psi) \rightarrow$ **patrové rovnice**



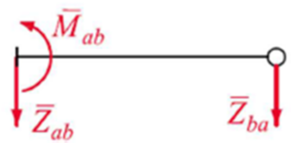
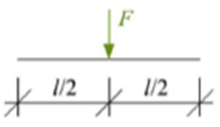
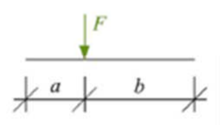
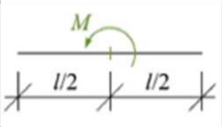
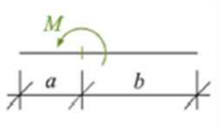
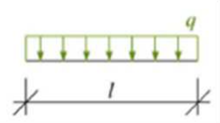



**Na zadané prutové konstrukci pomocí zjednodušené deformační metody vykreslete průběhy vnitřních sil**




$EI = \text{konst}$

# ZDM – KONCOVÉ MOMENTY/SÍLY PRUTU

|   |   | |   | |   | |
|---|---|---|---|---|---|---|
|   | $\bar{M}_{ab} = \frac{Fl}{8}$ | $\bar{M}_{ba} = -\frac{Fl}{8}$ | $\bar{M}_{ab} = \frac{3Fl}{16}$ | | | $\bar{M}_{ba} = -\frac{3Fl}{16}$ |
| | $\bar{Z}_{ab} = -\frac{F}{2}$ | $\bar{Z}_{ba} = -\frac{F}{2}$ | $\bar{Z}_{ab} = -\frac{11F}{16}$ | $\bar{Z}_{ba} = -\frac{5F}{16}$ | $\bar{Z}_{ab} = -\frac{5F}{16}$ | $\bar{Z}_{ba} = -\frac{11F}{16}$ |
|   | $\bar{M}_{ab} = \frac{F \cdot a \cdot b^2}{l^2}$ | $\bar{M}_{ba} = -\frac{F \cdot a^2 \cdot b}{l^2}$ | $\bar{M}_{ab} = \frac{F \cdot a \cdot b}{2l^2}(l+b)$ | | | $\bar{M}_{ba} = -\frac{F \cdot a \cdot b}{2l^2}(l+a)$ |
| | $\bar{Z}_{ab} = -\frac{F \cdot b^2}{l^3}(3l-2b)$ | $\bar{Z}_{ba} = -\frac{F \cdot a^2}{l^3}(3l-2a)$ | $\bar{Z}_{ab} = -\frac{F \cdot b}{2l^3}(3l^2-b^2)$ | $\bar{Z}_{ba} = -\frac{F \cdot a^2}{2l^3}(3l-a)$ | $\bar{Z}_{ab} = -\frac{F \cdot b^2}{2l^3}(3l-b)$ | $\bar{Z}_{ba} = -\frac{F \cdot a}{2l^3}(3l^2-a^2)$ |
|   | $\bar{M}_{ab} = \frac{M}{4}$ | $\bar{M}_{ba} = \frac{M}{4}$ | $\bar{M}_{ab} = \frac{M}{8}$ | | | $\bar{M}_{ba} = \frac{M}{8}$ |
| | $\bar{Z}_{ab} = -\frac{3M}{2l}$ | $\bar{Z}_{ba} = \frac{3M}{2l}$ | $\bar{Z}_{ab} = -\frac{9M}{8l}$ | $\bar{Z}_{ba} = \frac{9M}{8l}$ | $\bar{Z}_{ab} = -\frac{9M}{8l}$ | $\bar{Z}_{ba} = \frac{9M}{8l}$ |
|   | $\bar{M}_{ab} = \frac{M \cdot b}{l^2}(2l-3b)$ | $\bar{M}_{ba} = \frac{M \cdot a}{l^2}(2l-3a)$ | $\bar{M}_{ab} = \frac{M}{2l^2}(l^2-3b^2)$ | | | $\bar{M}_{ba} = \frac{M}{2l^2}(l^2-3a^2)$ |
| | $\bar{Z}_{ab} = -\frac{6M \cdot a \cdot b}{l^3}$ | $\bar{Z}_{ba} = \frac{6M \cdot a \cdot b}{l^3}$ | $\bar{Z}_{ab} = -\frac{3M}{2l^3}(l^2-b^2)$ | $\bar{Z}_{ba} = \frac{3M}{2l^3}(l^2-b^2)$ | $\bar{Z}_{ab} = -\frac{3M}{2l^3}(l^2-a^2)$ | $\bar{Z}_{ba} = \frac{3M}{2l^3}(l^2-a^2)$ |
|   | $\bar{M}_{ab} = \frac{q \cdot l^2}{12}$ | $\bar{M}_{ba} = -\frac{q \cdot l^2}{12}$ | $\bar{M}_{ab} = \frac{q \cdot l^2}{8}$ | | | $\bar{M}_{ba} = -\frac{q \cdot l^2}{8}$ |
| | $\bar{Z}_{ab} = -\frac{q \cdot l}{2}$ | $\bar{Z}_{ba} = -\frac{q \cdot l}{2}$ | $\bar{Z}_{ab} = -\frac{5q \cdot l}{8}$ | $\bar{Z}_{ba} = -\frac{3q \cdot l}{8}$ | $\bar{Z}_{ab} = -\frac{3q \cdot l}{8}$ | $\bar{Z}_{ba} = -\frac{5q \cdot l}{8}$ |
| Celkové koncové momenty prutu $k = \frac{2EI}{l}$ | $M_{ab} = \bar{M}_{ab} + k\left(2\varphi_a + \varphi_b + 3\frac{w_b - w_a}{l}\right)$ | | $M_{ab} = \bar{M}_{ab} + \frac{3}{4}k\left(2\varphi_a + 2\frac{w_b - w_a}{l}\right)$ | | | |
| | $M_{ba} = \bar{M}_{ba} + k\left(\varphi_a + 2\varphi_b + 3\frac{w_b - w_a}{l}\right)$ | | | | $M_{ba} = \bar{M}_{ba} + \frac{3}{4}k\left(2\varphi_b + 2\frac{w_b - w_a}{l}\right)$ | |
| Celkové koncové síly prutu | $Z_{ab} = \bar{Z}_{ab} - \frac{k}{l}\left(3\varphi_a + 3\varphi_b + 6\frac{w_b - w_a}{l}\right)$ | | $Z_{ab} = \bar{Z}_{ab} - \frac{3}{4}\frac{k}{l}\left(2\varphi_a + 2\frac{w_b - w_a}{l}\right)$ | | $Z_{ab} = \bar{Z}_{ab} - \frac{3}{4}\frac{k}{l}\left(2\varphi_b + 2\frac{w_b - w_a}{l}\right)$ | |
| | $Z_{ba} = \bar{Z}_{ba} + \frac{k}{l}\left(3\varphi_a + 3\varphi_b + 6\frac{w_b - w_a}{l}\right)$ | | $Z_{ba} = \bar{Z}_{ba} + \frac{3}{4}\frac{k}{l}\left(2\varphi_a + 2\frac{w_b - w_a}{l}\right)$ | | $Z_{ba} = \bar{Z}_{ba} + \frac{3}{4}\frac{k}{l}\left(2\varphi_b + 2\frac{w_b - w_a}{l}\right)$ | |

**Prut a–b**

$$M_{ba} = \bar{M}_{ba} + \frac{3}{4}k\left(2\varphi_b + 2\frac{w_b - w_a}{l}\right)$$

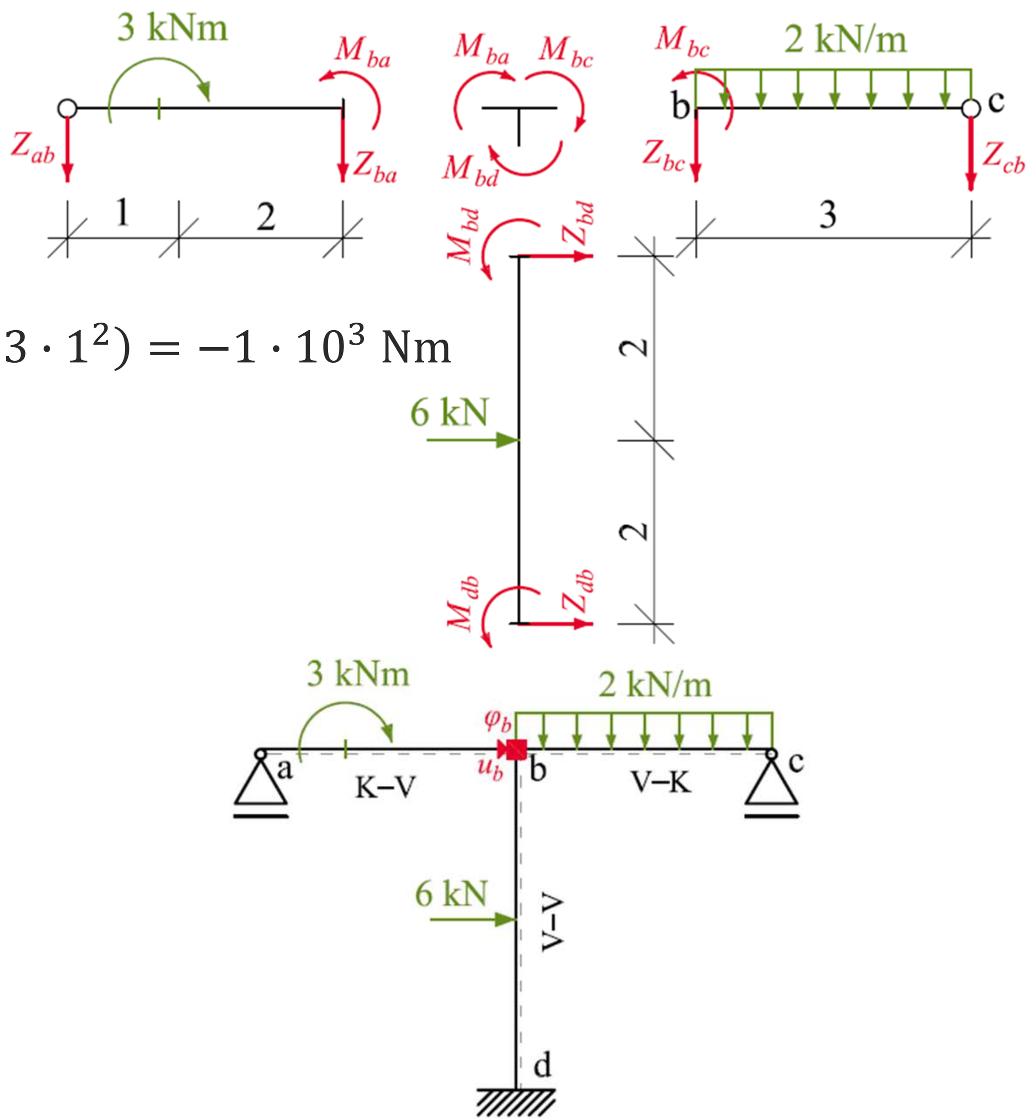
$$\bar{M}_{ba} = \frac{M}{2l^2}(l^2 - 3a^2) = \frac{-3\cdot 10^3}{2\cdot 3^2}(3^2 - 3\cdot 1^2) = -1\cdot 10^3\ \text{Nm}$$

$$k_{ab} = \frac{2EI}{l} = \frac{2EI}{3}$$

$$w_b = w_a = 0$$

$$M_{ba} = -1\cdot 10^3 + \frac{3}{4}\cdot\frac{2EI}{3}(2\varphi_b) \rightarrow$$

$$\boldsymbol{M_{ba} = -1\cdot 10^3 + EI\cdot\varphi_b}$$



## Prut b–c

$$M_{bc} = \bar{M}_{bc} + \frac{3}{4} k \left( 2\varphi_b + 2\frac{w_c - w_b}{l} \right)$$

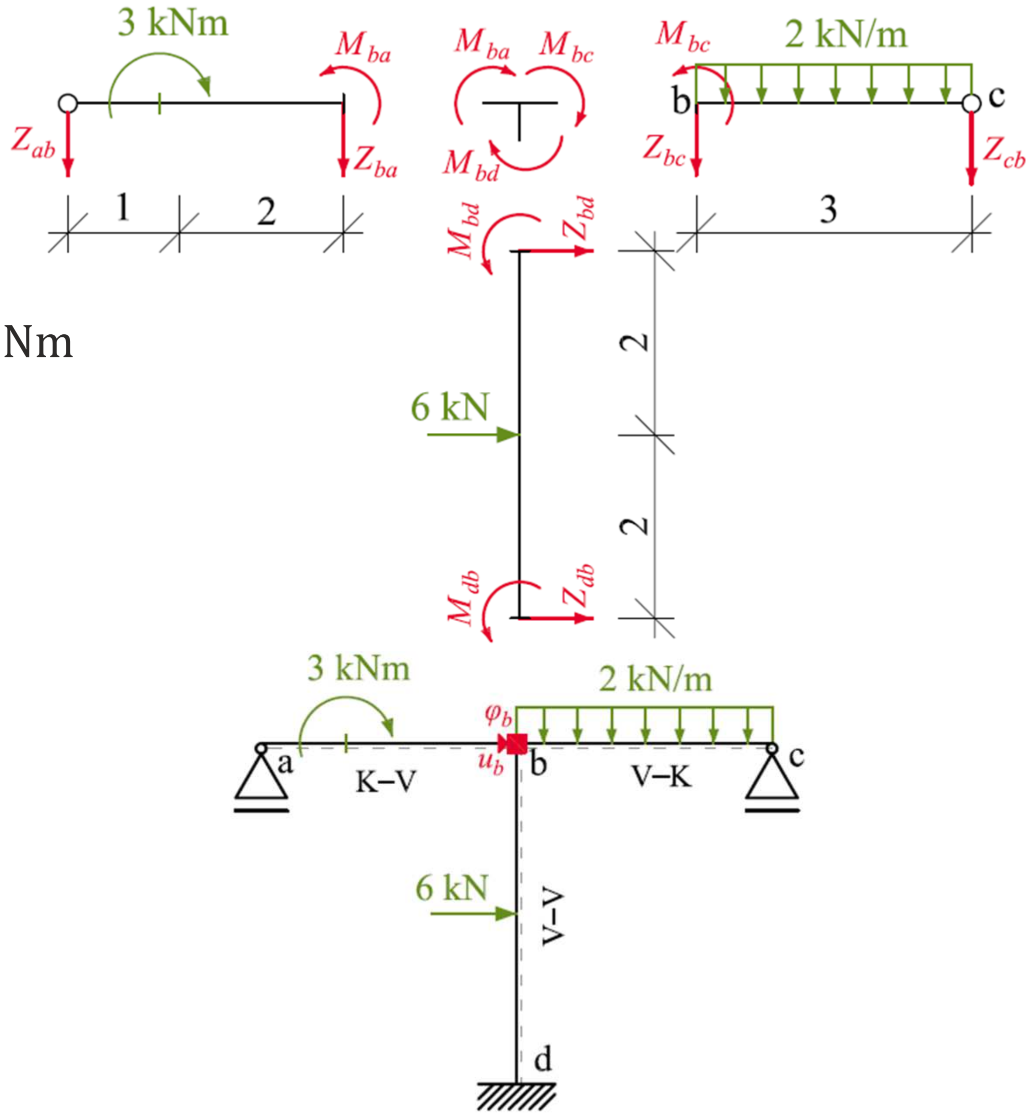
$$\bar{M}_{bc} = \frac{q \cdot l^2}{8} = \frac{2 \cdot 10^3 \cdot 3^2}{8} = 2{,}25 \cdot 10^3 \text{ Nm}$$

$$k_{bc} = \frac{2EI}{l} = \frac{2EI}{3}$$

$$w_c = w_b = 0$$

$$M_{bc} = 2{,}25 \cdot 10^3 + \frac{3}{4} \cdot \frac{2EI}{3} (2\varphi_b) \rightarrow$$

$$\boldsymbol{M_{bc} = 2{,}25 \cdot 10^3 + EI \cdot \varphi_b}$$



# ZDM – ROVINNÝ RÁM

## Prut d–b

$$M_{bd} = \bar{M}_{bd} + k\left(\varphi_d + 2\varphi_b + 3\frac{w_b - w_d}{l}\right)$$

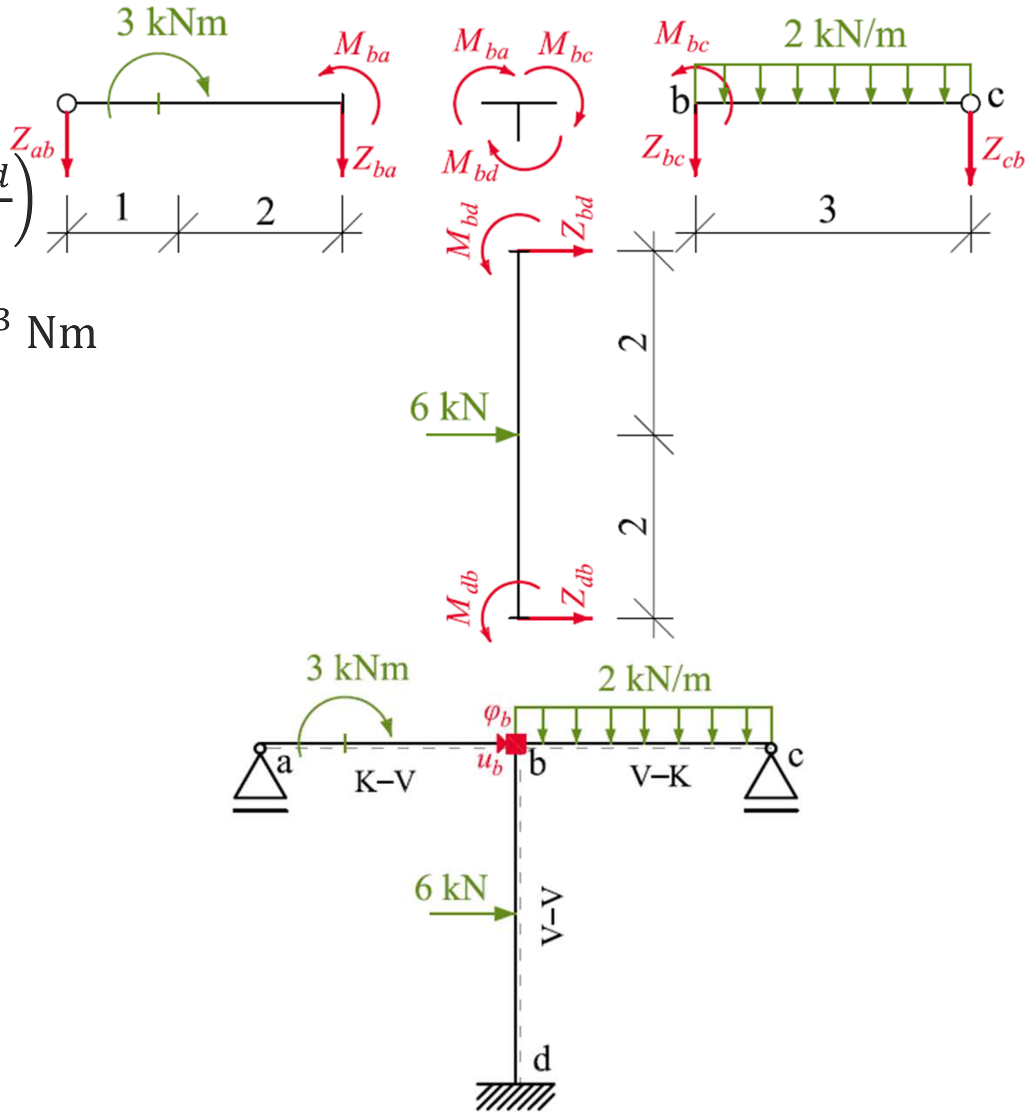
$$\bar{M}_{bd} = -\frac{F \cdot l}{8} = -\frac{6 \cdot 10^3 \cdot 4}{8} = -3 \cdot 10^3 \text{ Nm}$$

$$k_{db} = \frac{2EI}{l} = \frac{2EI}{4} = \frac{EI}{2}$$

$$\varphi_d = 0; w_d = 0$$

$$M_{bd} = -3 \cdot 10^3 + \frac{EI}{2}\left(2\varphi_b + 3\frac{u}{4}\right) \rightarrow$$

$$\boldsymbol{M_{bd} = -3 \cdot 10^3 + EI \cdot \varphi_b + \frac{3}{8}EI \cdot u}$$



# ZDM – ROVINNÝ RÁM

## Styčníková rovnice

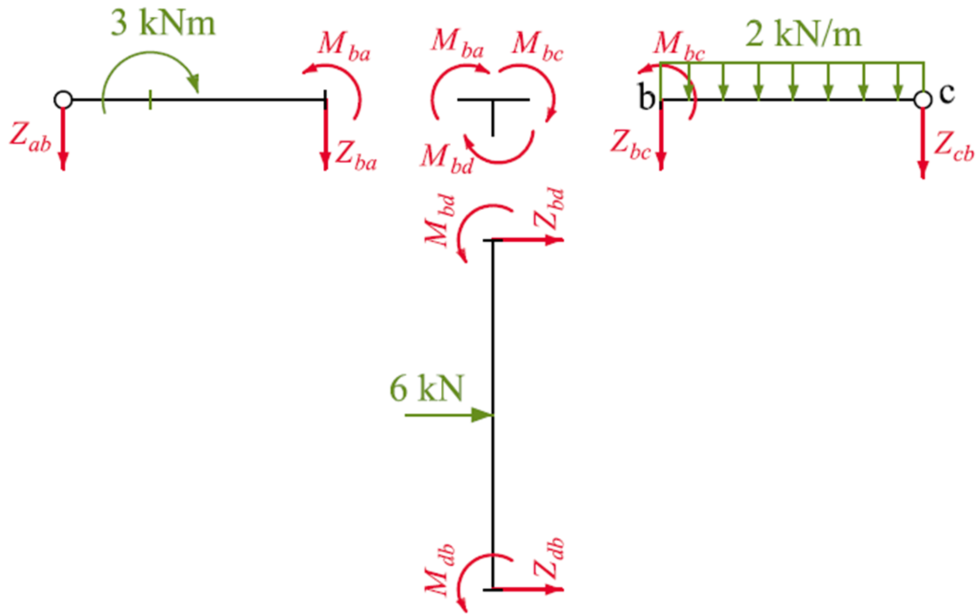
- momentová podmínka rovnováhy

$$\varphi_b\colon\ M_{ba} + M_{bc} + M_{bd} = 0$$

$$M_{ba} = -1 \cdot 10^3 + EI \cdot \varphi_b$$

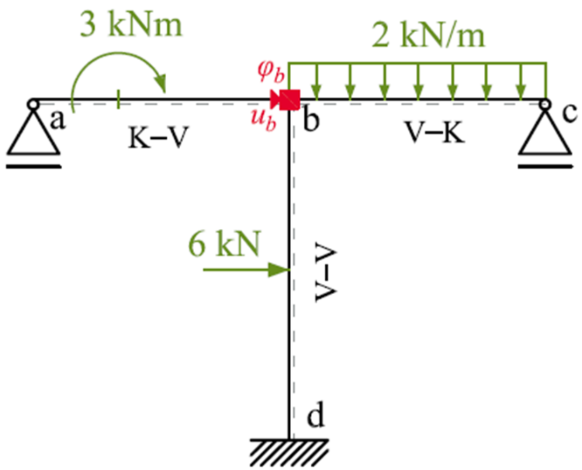
$$M_{bc} = 2{,}25 \cdot 10^3 + EI \cdot \varphi_b$$

$$M_{bd} = -3 \cdot 10^3 + EI \cdot \varphi_b + \frac{3}{8} EI \cdot u$$

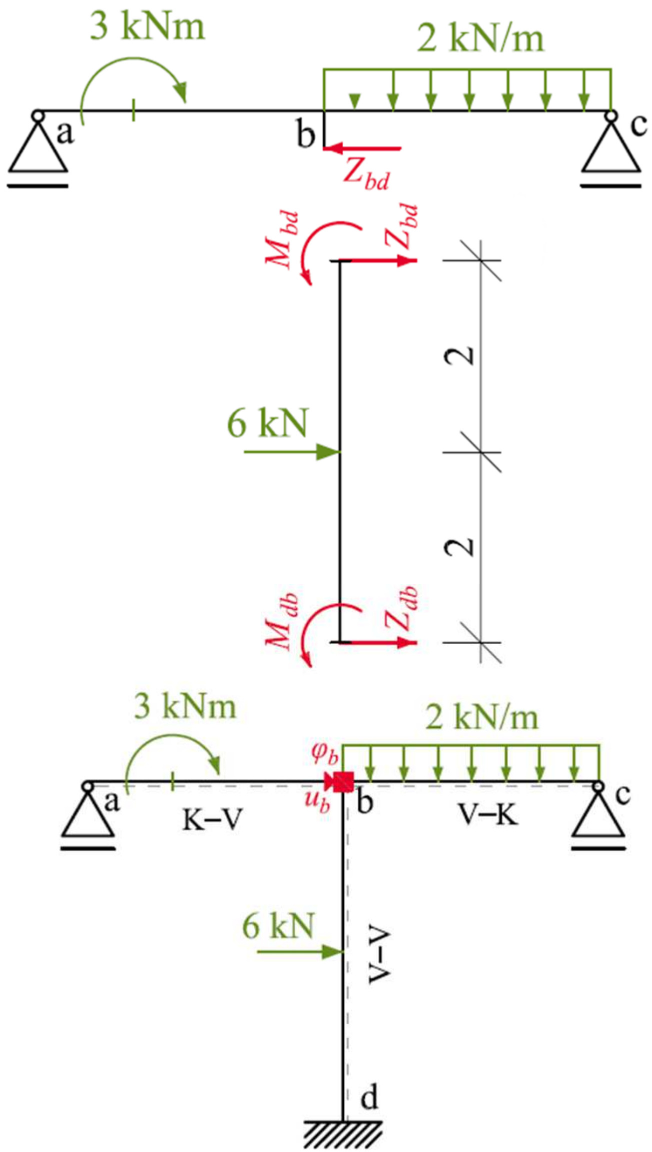



$$-1 \cdot 10^3 + EI \cdot \varphi_b + 2{,}25 \cdot 10^3 + EI \cdot \varphi_b +$$
$$+ \left(-3 \cdot 10^3 + EI \cdot \varphi_b + \frac{3}{8} EI \cdot u\right) = 0$$

$$\boldsymbol{-1{,}75 \cdot 10^3 + 3EI \cdot \varphi_b + \frac{3}{8} EI \cdot u = 0}$$



# ZDM – ROVINNÝ RÁM




## Patrová rovnice

- součtová podmínka rovnováhy

$$u_b\colon\ Z_{bd} = 0$$

$$Z_{bd} = \bar{Z}_{bd} + \frac{k}{l}\left(3\varphi_d + 3\varphi_b + 6\,\frac{w_b - w_d}{l}\right)$$

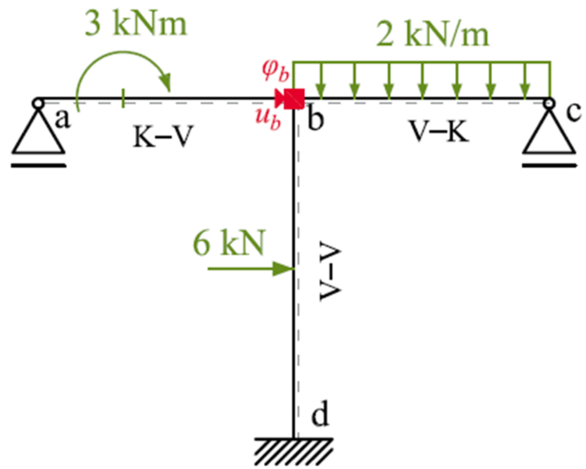
$$\bar{Z}_{bd} = -\frac{F}{2} = -\frac{6\cdot 10^3}{2} = -3\cdot 10^3$$

$$k_{bd} = \frac{2EI}{l} = \frac{2EI}{4} = \frac{EI}{2}$$

$$\varphi_d = 0; w_d = 0$$

$$Z_{bd} = -3\cdot 10^3 + \frac{EI}{2\cdot 4}\left(3\varphi_b + 6\,\frac{u}{4}\right) \rightarrow$$

$$\boldsymbol{Z_{bd} = -3\cdot 10^3 + \frac{3}{8}EI\cdot\varphi_b + \frac{3}{16}EI\cdot u = 0}$$




## Soustava rovnic

$$\varphi_b\colon\ -1{,}75\cdot 10^3 + 3EI\cdot\varphi_b + \frac{3}{8}EI\cdot u = 0$$

$$u_b\colon\ -3\cdot 10^3 + \frac{3}{8}EI\cdot\varphi_b + \frac{3}{16}EI\cdot u = 0$$

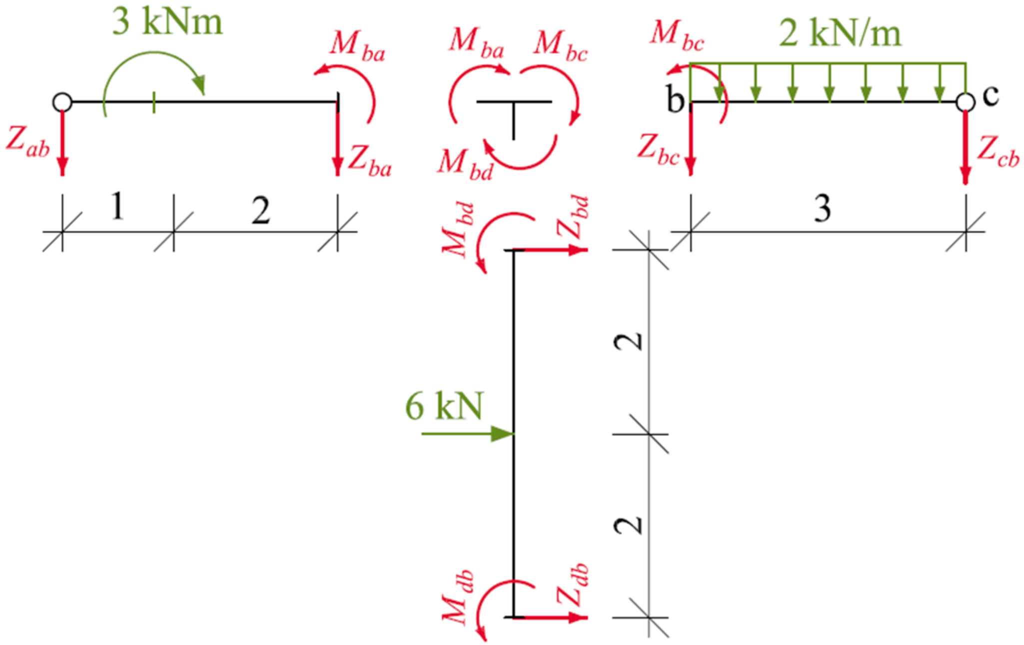
$$3EI\cdot\varphi_b + \tfrac{3}{8}EI\cdot u = 1{,}75\cdot 10^3\ /\cdot\left(-\tfrac{1}{2}\right)$$

$$\frac{3}{8}EI\cdot\varphi_b + \frac{3}{16}EI\cdot u = 3\cdot 10^3$$

$$-\frac{9}{8}EI\cdot\varphi_b = \frac{17}{8}\cdot 10^3 \rightarrow \boldsymbol{\varphi_b} = -\frac{\mathbf{17}}{\mathbf{9}\boldsymbol{EI}}\cdot\mathbf{10^3\ rad}\ \left(\frac{-1,\bar{8}\cdot 10^3}{EI}\,\mathrm{rad}\right)$$

$$\frac{3}{8}EI\cdot\left(-\frac{17}{9EI}\cdot 10^3\right) + \frac{3}{16}EI\cdot u = 3\cdot 10^3 \rightarrow \boldsymbol{u} = \frac{\mathbf{178}}{\mathbf{9}\boldsymbol{EI}}\cdot\mathbf{10^3\ m}\left(\frac{19,\bar{7}\cdot 10^3}{EI}\,\mathrm{m}\right)$$

# ZDM – ROVINNÝ RÁM




## Prut a–b

$$M_{ba} = -1 \cdot 10^3 + EI \cdot \varphi_b =$$

$$= -1 \cdot 10^3 + EI \cdot \left(-\frac{17}{9EI} \cdot 10^3\right) \rightarrow$$

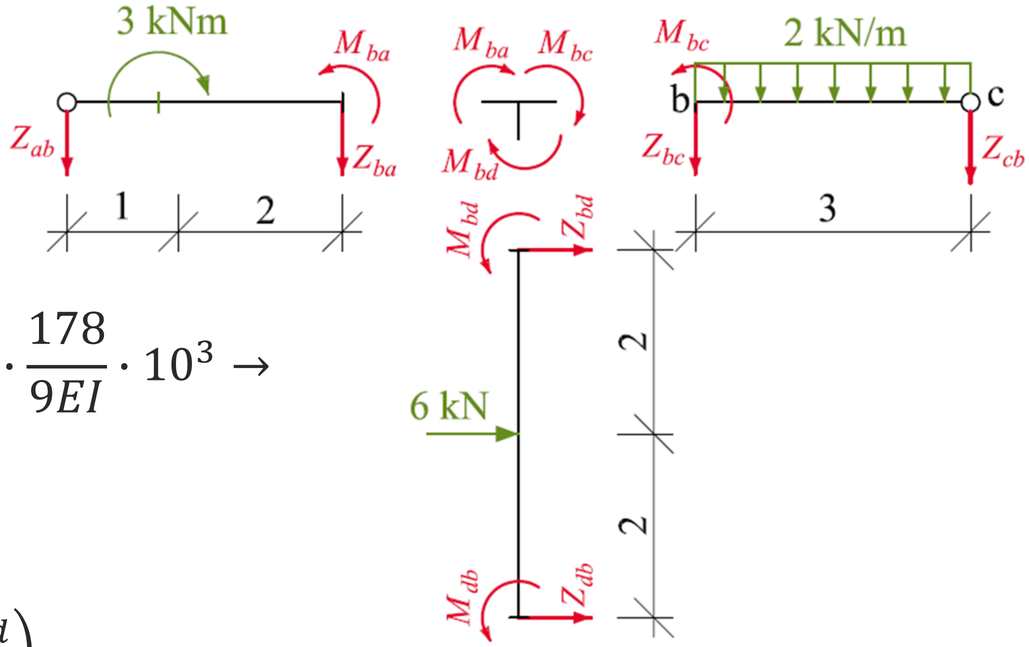
$$\boldsymbol{M_{ba}} = -2,\overline{8} \cdot 10^3 \text{ Nm} = \mathbf{-2,\overline{8}\ kNm}$$

## Prut b–c

$$M_{bc} = 2{,}25 \cdot 10^3 + EI \cdot \varphi_b = 2{,}25 \cdot 10^3 + EI \cdot \left(-\frac{17}{9EI} \cdot 10^3\right)$$

$$\boldsymbol{M_{bc}} = 0{,}36\overline{1} \cdot 10^3 \text{ Nm} = \mathbf{0,36\overline{1}\ kNm}$$

# ZDM – ROVINNÝ RÁM




## Prut d–b

$$M_{bd} = -3 \cdot 10^3 + EI \cdot \varphi_b + \frac{3}{8} EI \cdot u$$

$$= -3 \cdot 10^3 + EI \cdot \left(-\frac{17}{9EI} \cdot 10^3\right) + \frac{3}{8} EI \cdot \frac{178}{9EI} \cdot 10^3 \rightarrow$$

$$\boldsymbol{M_{bd}} = 2{,}52\overline{7} \cdot 10^3 \text{ Nm} = \mathbf{2,52\overline{7}\ kNm}$$

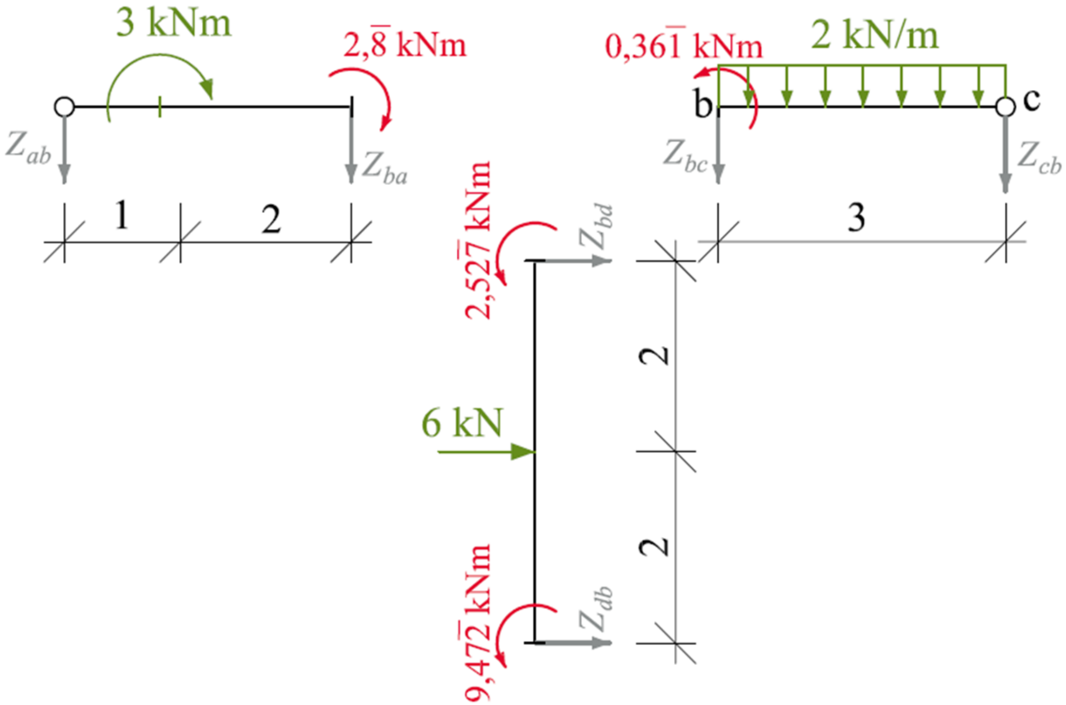
$$M_{db} = \bar{M}_{db} + k\left(2\varphi_d + \varphi_b + 3\frac{w_b - w_d}{l}\right)$$

$$\bar{M}_{db} = \frac{F \cdot l}{8} = \frac{6 \cdot 10^3 \cdot 4}{8} = 3 \cdot 10^3 \text{ Nm}$$

$$M_{db} = 3 \cdot 10^3 + \frac{EI}{2}\left(-\frac{17}{9EI} \cdot 10^3 + 3 \cdot \frac{\frac{178}{9EI} \cdot 10^3}{4}\right) \rightarrow$$

$$\boldsymbol{M_{db}} = 9{,}47\overline{2} \cdot 10^3 \text{ Nm} = \mathbf{9,47\overline{2}\ kNm}$$

# ZDM – ROVINNÝ RÁM




## Prut a–b

$$Z_{ab} = -Z_{ba} = \frac{3 + 2,\bar{8}}{3} = 1,\overline{962}\ \text{kN}$$

## Prut b–c

$$Z_{bc} = -\frac{0{,}36\bar{1}}{3} - \frac{2 \cdot 3}{2} = 3{,}12\ \text{kN}$$

$$Z_{cb} = \frac{0{,}36\bar{1}}{3} - \frac{2 \cdot 3}{2} = -2{,}88\ \text{kN}$$

## Prut d–b

$$Z_{db} = -\frac{9{,}47\bar{2} + 2{,}52\bar{7}}{4} - \frac{6}{2} = -6\ \text{kN}$$

$$Z_{bd} = \frac{9{,}47\bar{2} + 2{,}52\bar{7}}{4} - \frac{6}{2} = 0\ \text{kN}$$

# ZDM – ROVINNÝ RÁM