BDA016 Stavební mechanika 2
7. přednáška

- Příhradový nosník řešený silovou metodou

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V přednášce jsou použity obrázky z učebnice Kadlčák, J., Kytýr, J. Statika stavebních konstrukcí II. Staticky neurčité prutové konstrukce. Nakladatelství VUTIUM v Brně, 2004.

PŘÍHRADOVÝ NOSNÍK

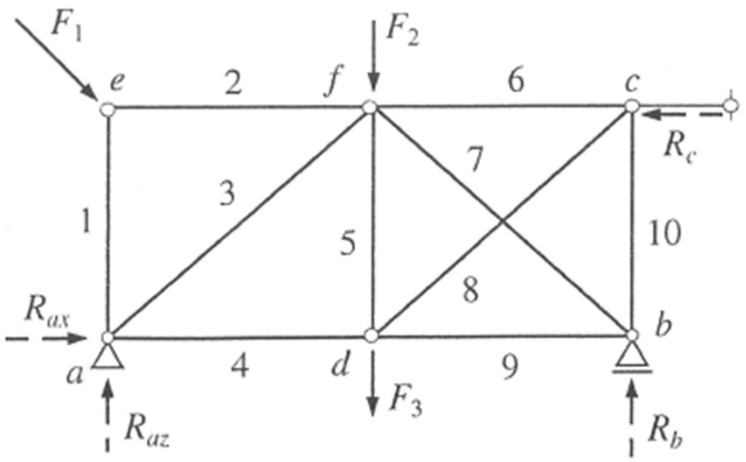
Stupeň statické neurčitosti

- $n_s = r - m$
- $n_s = p + a - 2b$

p – počet prutu

a – počet vnějších vazeb

b – počet styčníků



Stupeň statické neurčitosti – zevní

- $n_{s,ext} = r_{ext} - m_{ext} = a - 3$

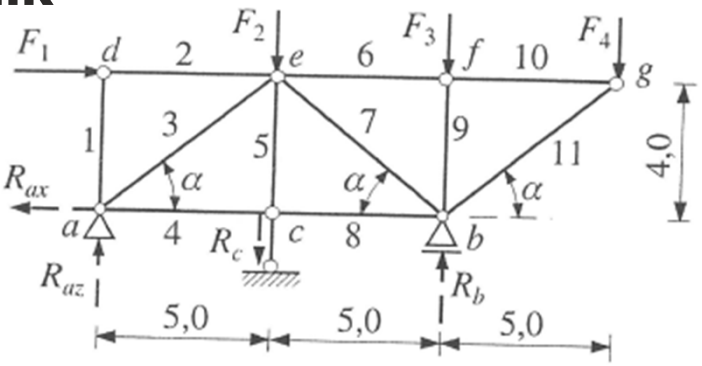
Stupeň statické neurčitosti – vnitřní

- $n_{s,vnitřní} = p + 3 - 2b$

PŘÍHRADOVÝ NOSNÍK

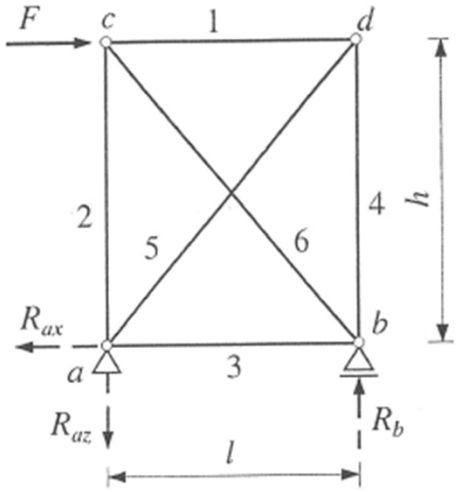
Zevně staticky neurčitý příhradový nosník

- $n_s = r - m = (4 + 30) - 33 = 1$
- $n_s = p + a - 2b = 11 + 4 - 2 \cdot 7 = 1$
- $n_{s,ext} = r_{ext} - m_{ext} = a - 3 = 4 - 3 = 1$
- $n_{s,vnitřní} = p + 3 - 2b = 11 + 3 - 2 \cdot 7 = 0$



Vnitřně staticky neurčitý příhradový nosník

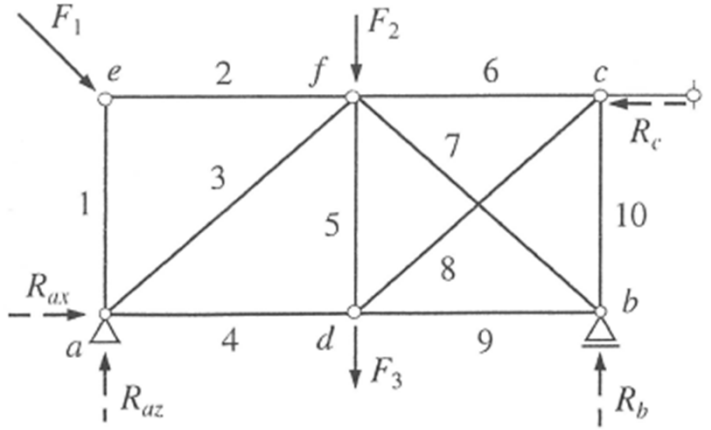
- $n_s = r - m = (3 + 16) - 18 = 1$
- $n_s = p + a - 2b = 6 + 3 - 2 \cdot 4 = 1$
- $n_{s,ext} = r_{ext} - m_{ext} = a - 3 = 3 - 3 = 0$
- $n_{s,vnitřní} = p + 3 - 2b = 6 + 3 - 2 \cdot 4 = 1$



PŘÍHRADOVÝ NOSNÍK

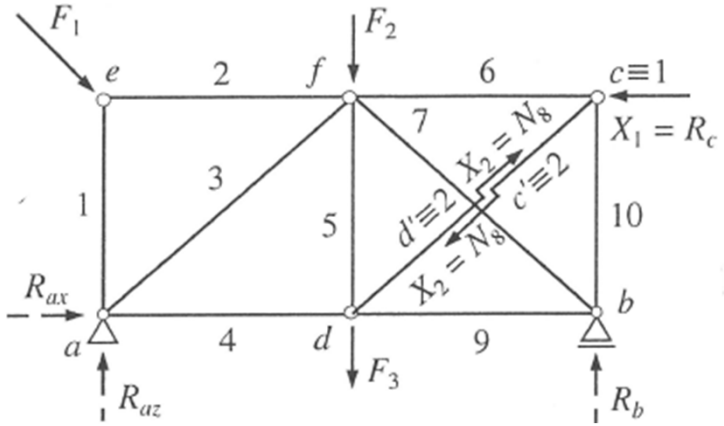
Zevně i vnitřně staticky neurčitý příhradový nosník

- $n_s = r - m = (4 + 28) - 30 = 2$
- $n_s = p + a - 2b = 10 + 4 - 2 \cdot 6 = 2$
- $n_{s,ext} = r_{ext} - m_{ext} = a - 3 = 4 - 3 = 1$
- $n_{s,vnitřní} = p + 3 - 2b = 10 + 3 - 2 \cdot 6 = 1$



Základní soustava – SU

- odebrání 2 vazeb – 1 vnitřní a 1 vnější



Staticky neurčité veličiny

- $X_1 = R_{c,x}$
- $X_2 = N_8$

Deformační podmínky

- $\delta_1 = u_c = 0; \qquad \delta_2 = \delta_{c'd'} = 0$

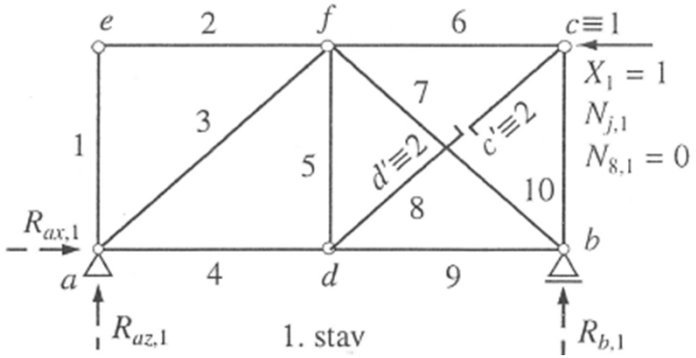
PŘÍHRADOVÝ NOSNÍK

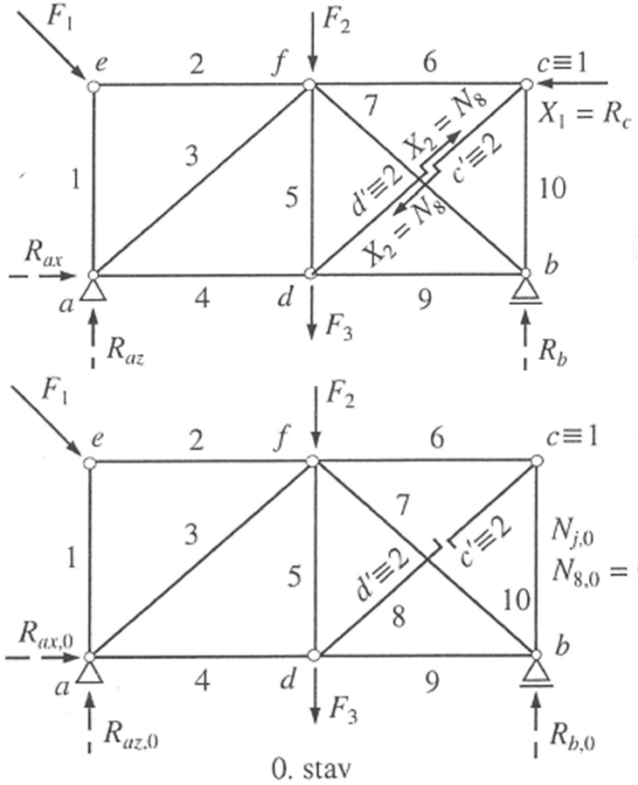
Princip superpozice
→ 3 zatěžovací stavy

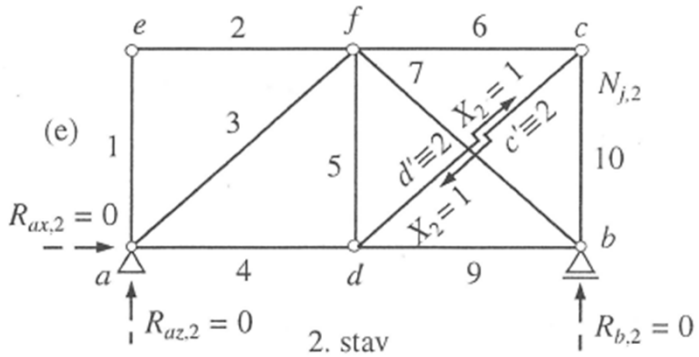
- 0. ZS – zatížení
- 1. ZS – $X_1 = R_{c,x} = 1$
- 2. ZS – $X_2 = N_8 = 1$

Systém lineárních rovnic

- $\delta_1 = \delta_{10} + \delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 = 0$
- $\delta_2 = \delta_{20} + \delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 = 0$







PŘÍHRADOVÝ NOSNÍK

Deformační součinitele

→ z principu virtuálních prací

pro zadané zatížení (silové + teplotní)

- $\delta_{i0} = \int_0^s \frac{N_i N_0}{EA} \mathrm{d}s + \int_0^s N_i \alpha_t \Delta T_0 \mathrm{d}s = \sum_{j=1}^{p} \frac{N_{ji} \cdot N_{j0} \cdot l_j}{EA_j} + \alpha_t \sum_{j=1}^{p} N_{ji} \cdot \Delta T_{0j} \cdot l_j$

pro popuštění podpor

- $\delta_{i0,p} = -\sum_{r=1}^{p_v} R_{r,i} \cdot \delta_r$ → stejný postup jako v případě rámových konstrukcí

pro jednotkové zatížení

- $\delta_{ik} = \int_0^s \frac{N_i N_k}{EA} \mathrm{d}s = \sum_{j=1}^{p} \frac{N_{ji} \cdot N_{jk} \cdot l_j}{EA_j}$
- $i = 1, 2, \cdots, n_s$
- $\delta_{ik} = \delta_{ki}$

SILOVÁ METODA

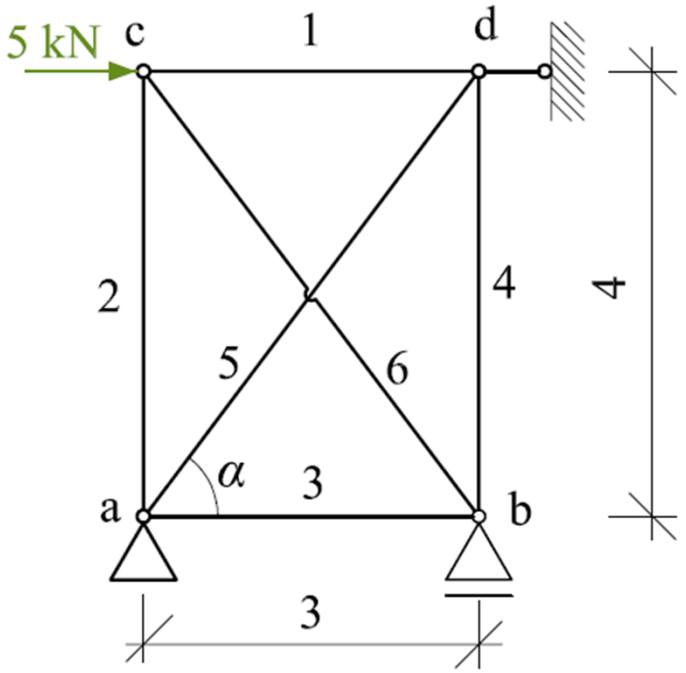
Na dané příhradové konstrukci pomocí silové metody určete osové síly v jednotlivých prutech.

$E = \text{konst}$

$A = \text{konst}$

$\cos\alpha = \frac{3}{5} = 0{,}6$

$\sin\alpha = \frac{4}{5} = 0{,}8$



SILOVÁ METODA

Stupeň statické neurčitosti

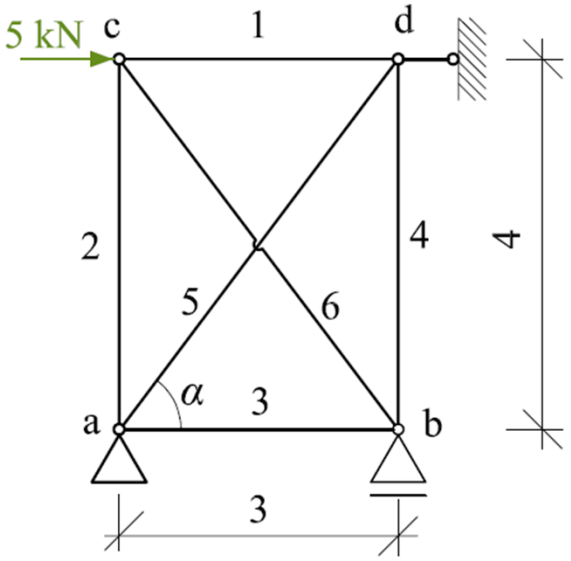
- $n_s = r - m = (4 + 16) - 18 = 2$
- $n_s = p + a - 2b = 6 + 4 - 2 \cdot 4 = 2$
- $n_{s,ext} = r_{ext} - m_{ext} = a - 3 = 4 - 3 = 1$
- $n_{s,vnitřní} = p + 3 - 2b = 6 + 3 - 2 \cdot 4 = 1$

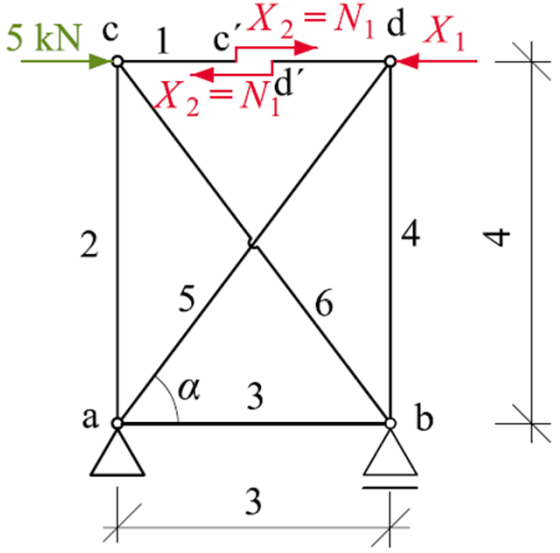
Základní soustava – staticky určitá

- odebereme n_s přebytečných vazeb
- nahradíme staticky neurčitými veličinami

Deformační podmínky

- $\delta_1 = u_d = 0; \quad \delta_{10} + \delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 = 0$
- $\delta_2 = \delta_{c'd'} = 0; \; \delta_{20} + \delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 = 0$

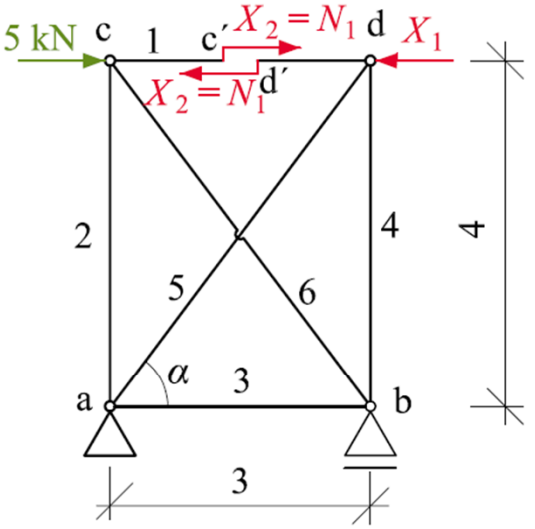


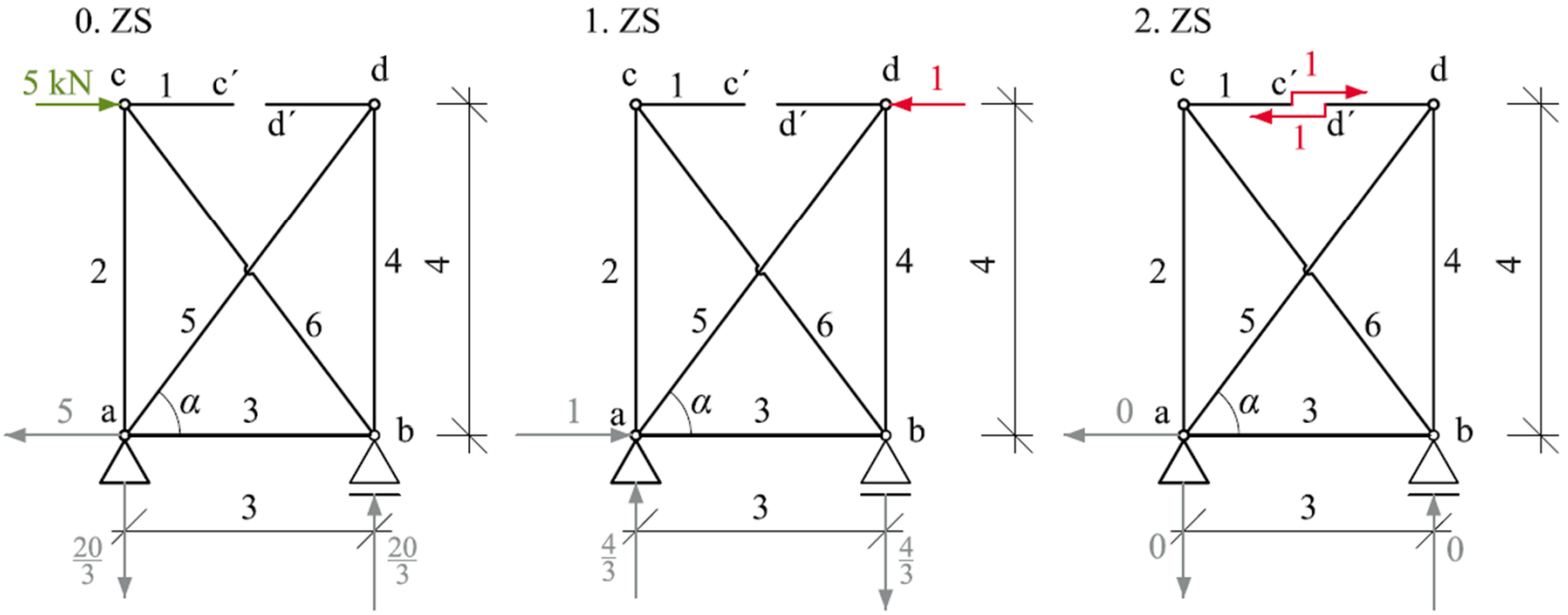


SILOVÁ METODA

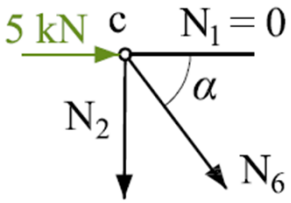
Zatěžovací stavy – princip superpozice

- $\delta_{10} + \delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 = 0$
- $\delta_{20} + \delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 = 0$
- $\delta_{i0} = \frac{1}{EA} \sum_{j=1}^{p} N_{ji} \cdot N_{j0} \cdot l_j \; ; \; \delta_{ik} = \frac{1}{EA} \sum_{j=1}^{p} N_{ji} \cdot N_{jk} \cdot l_j$

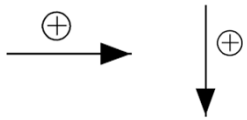




SILOVÁ METODA



0. ZS



styčník „c"

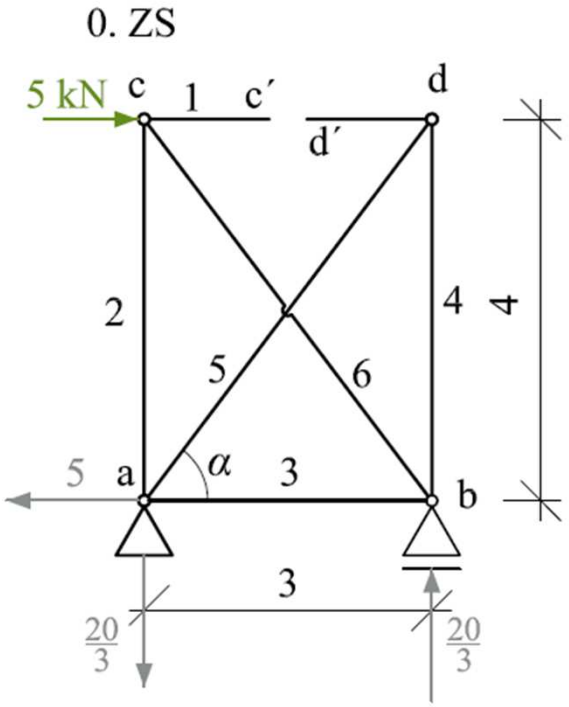
- $\sum F_{i,x} = 0;\ 5 + \cos\alpha \cdot N_6 = 0 \rightarrow \boldsymbol{N_6 = -\frac{25}{3}\ \mathbf{kN}}$
- $\sum F_{i,z} = 0;\ N_2 + \sin\alpha \cdot N_6 = 0 \rightarrow \boldsymbol{N_2 = \frac{20}{3}\mathbf{kN}}$

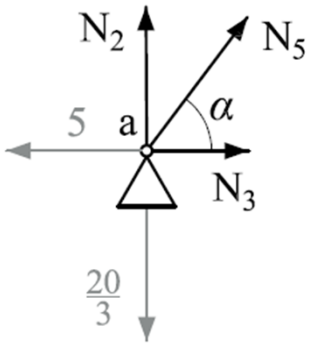
styčník „a"

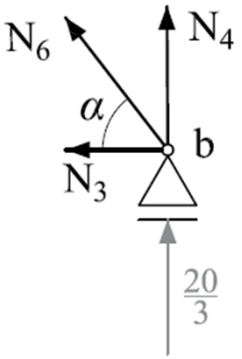
- $\sum F_{i,z} = 0;\ \frac{20}{3} - N_2 - \sin\alpha \cdot N_5 = 0 \rightarrow \boldsymbol{N_5 = 0}$
- $\sum F_{i,x} = 0;\ -5 + N_3 + \cos\alpha \cdot N_5 = 0 \rightarrow \boldsymbol{N_3 = 5\ \mathbf{kN}}$

styčník „b"

- $\sum F_{i,z} = 0;\ -\frac{20}{3} - N_4 - \sin\alpha \cdot N_6 = 0 \rightarrow \boldsymbol{N_4 = 0}$
- $\sum F_{i,x} = 0;\ -N_3 - \cos\alpha \cdot N_6 = 0 \rightarrow 0 = 0$

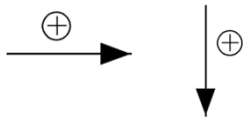






SILOVÁ METODA

1. ZS



styčník „d"

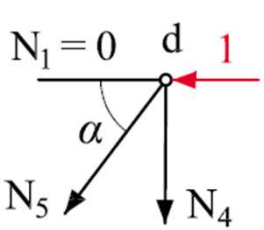
- $\sum F_{i,x} = 0;\ -1 - \cos\alpha \cdot N_5 = 0 \rightarrow \boldsymbol{N_5 = -\frac{5}{3}}$
- $\sum F_{i,z} = 0;\ N_4 + \sin\alpha \cdot N_5 = 0 \rightarrow \boldsymbol{N_4 = \frac{4}{3}}$

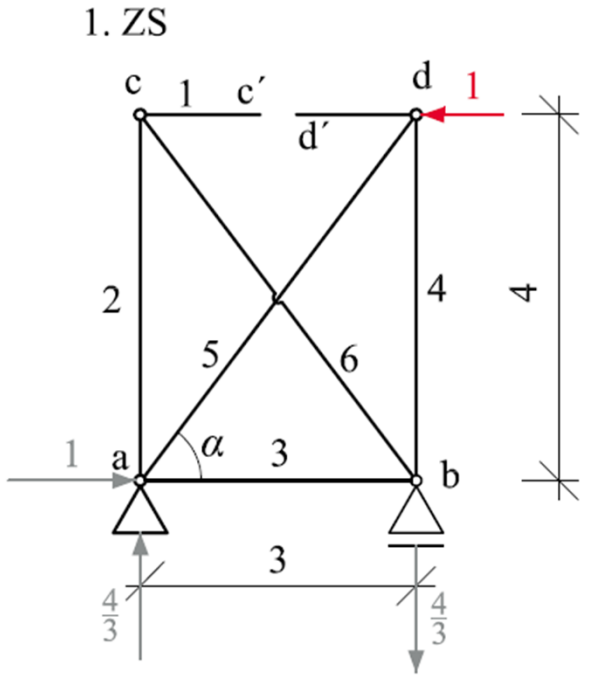
styčník „c"

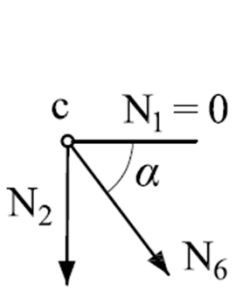
- $\sum F_{i,x} = 0;\ \cos\alpha \cdot N_6 = 0 \rightarrow \boldsymbol{N_6 = 0}$
- $\sum F_{i,z} = 0;\ N_2 + \sin\alpha \cdot N_6 = 0 \rightarrow \boldsymbol{N_2 = 0}$

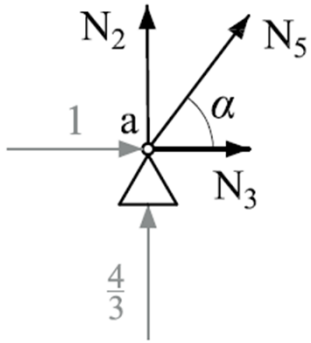
styčník „a"

- $\sum F_{i,x} = 0;\ 1 + N_3 + \cos\alpha \cdot N_5 = 0 \rightarrow \boldsymbol{N_3 = 0}$
- $\sum F_{i,z} = 0; -\frac{4}{3} - N_2 - \sin\alpha \cdot N_5 = 0 \rightarrow 0 = 0$



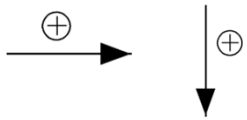


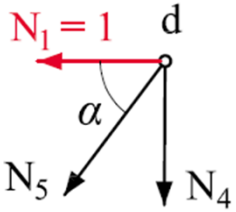




SILOVÁ METODA

2. ZS





styčník „d"

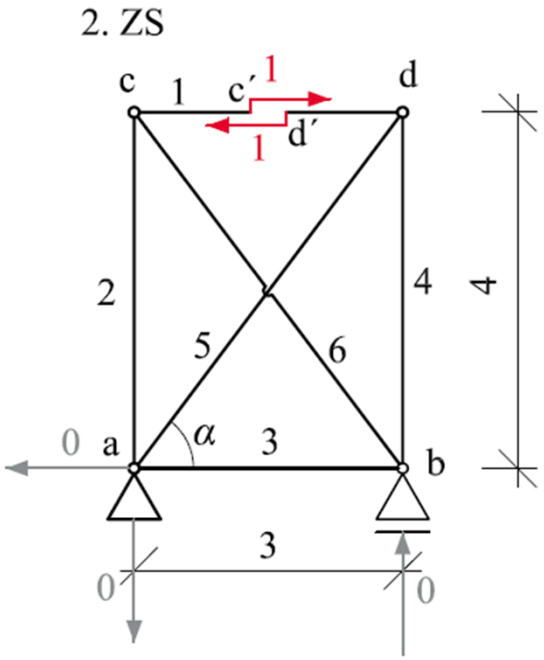
- $\sum F_{i,x} = 0;\ -1 - \cos\alpha \cdot N_5 = 0 \rightarrow \boldsymbol{N_5 = -\frac{5}{3}}$
- $\sum F_{i,z} = 0;\ N_4 + \sin\alpha \cdot N_5 = 0 \rightarrow \boldsymbol{N_4 = \frac{4}{3}}$

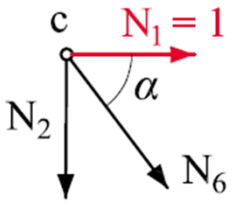
styčník „c"

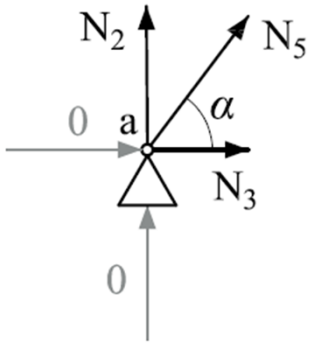
- $\sum F_{i,x} = 0;\ 1 + \cos\alpha \cdot N_6 = 0 \rightarrow \boldsymbol{N_6 = -\frac{5}{3}}$
- $\sum F_{i,z} = 0;\ N_2 + \sin\alpha \cdot N_6 = 0 \rightarrow \boldsymbol{N_2 = \frac{4}{3}}$

styčník „a"

- $\sum F_{i,x} = 0;\ N_3 + \cos\alpha \cdot N_5 = 0 \rightarrow \boldsymbol{N_3 = 1}$
- $\sum F_{i,z} = 0; -N_2 - \sin\alpha \cdot N_5 = 0 \rightarrow 0 = 0$







SILOVÁ METODA

- $\delta_{10} + \delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 = 0$ $\qquad \delta_{i0} = \frac{1}{EA} \sum_{j=1}^{p} N_{ji} \cdot N_{j0} \cdot l_j$
- $\delta_{20} + \delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 = 0$ $\qquad \delta_{ik} = \frac{1}{EA} \sum_{j=1}^{p} N_{ji} \cdot N_{jk} \cdot l_j$

prut j	l_j [m]	N_{0j} [N]	N_{1j} [−]	N_{2j} [−]	$N_{1j}N_{0j}l_j$ [Nm]	$N_{1j}N_{1j}l_j$ [m]	$N_{1j}N_{2j}l_j$ [m]	$N_{2j}N_{0j}l_j$ [Nm]	$N_{2j}N_{1j}l_j$ [m]	$N_{2j}N_{2j}l_j$ [m]
1	3	0	0	1	0	0	0	0	0	3
2	4	$\frac{20}{3} \cdot 10^3$	0	$\frac{4}{3}$	0	0	0	$\frac{320}{9} \cdot 10^3$	0	$\frac{64}{9}$
3	3	$5 \cdot 10^3$	0	1	0	0	0	$15 \cdot 10^3$	0	3
4	4	0	$\frac{4}{3}$	$\frac{4}{3}$	0	$\frac{64}{9}$	$\frac{64}{9}$	0	$\frac{64}{9}$	$\frac{64}{9}$
5	5	0	$-\frac{5}{3}$	$-\frac{5}{3}$	0	$\frac{125}{9}$	$\frac{125}{9}$	0	$\frac{125}{9}$	$\frac{125}{9}$
6	5	$-\frac{25}{3} \cdot 10$	0	$-\frac{5}{3}$	0	0	0	$\frac{625}{9} \cdot 10^3$	0	$\frac{125}{9}$
Σ					**0**	**21**	**21**	**$120 \cdot 10^3$**	**21**	**48**

SILOVÁ METODA

$$\delta_{i0} = \frac{1}{EA}\sum_{j=1}^{p} N_{ji} \cdot N_{j0} \cdot l_j \; ; \qquad \delta_{ik} = \frac{1}{EA}\sum_{j=1}^{p} N_{ji} \cdot N_{jk} \cdot l_j$$

$$\delta_{10} + \delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 = 0$$
$$\delta_{20} + \delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 = 0$$

$$\frac{0}{EA} + \frac{21}{EA} \cdot X_1 + \frac{21}{EA} \cdot X_2 = 0 \rightarrow \qquad 21 \cdot X_1 + 21 \cdot X_2 = 0$$

$$\frac{120 \cdot 10^3}{EA} + \frac{21}{EA} \cdot X_1 + \frac{48}{EA} \cdot X_2 = 0 \rightarrow \qquad 21 \cdot X_1 + 48 \cdot X_2 = -120 \cdot 10^3$$

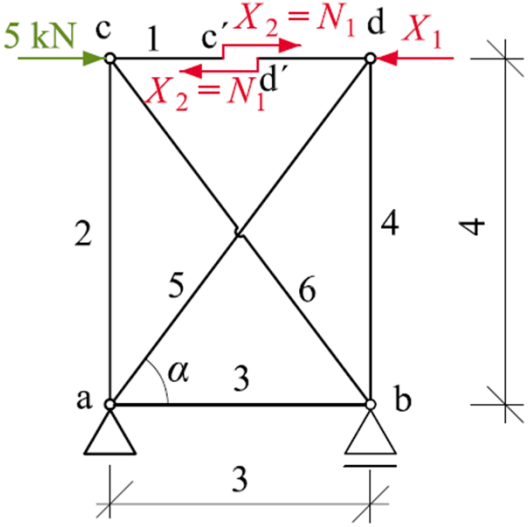
$$27 \cdot X_2 = -120 \cdot 10^3 \rightarrow \boldsymbol{X_2} = -4,\overline{4} \cdot 10^3 \text{ N} = \mathbf{-4,\overline{4}\ kN}$$

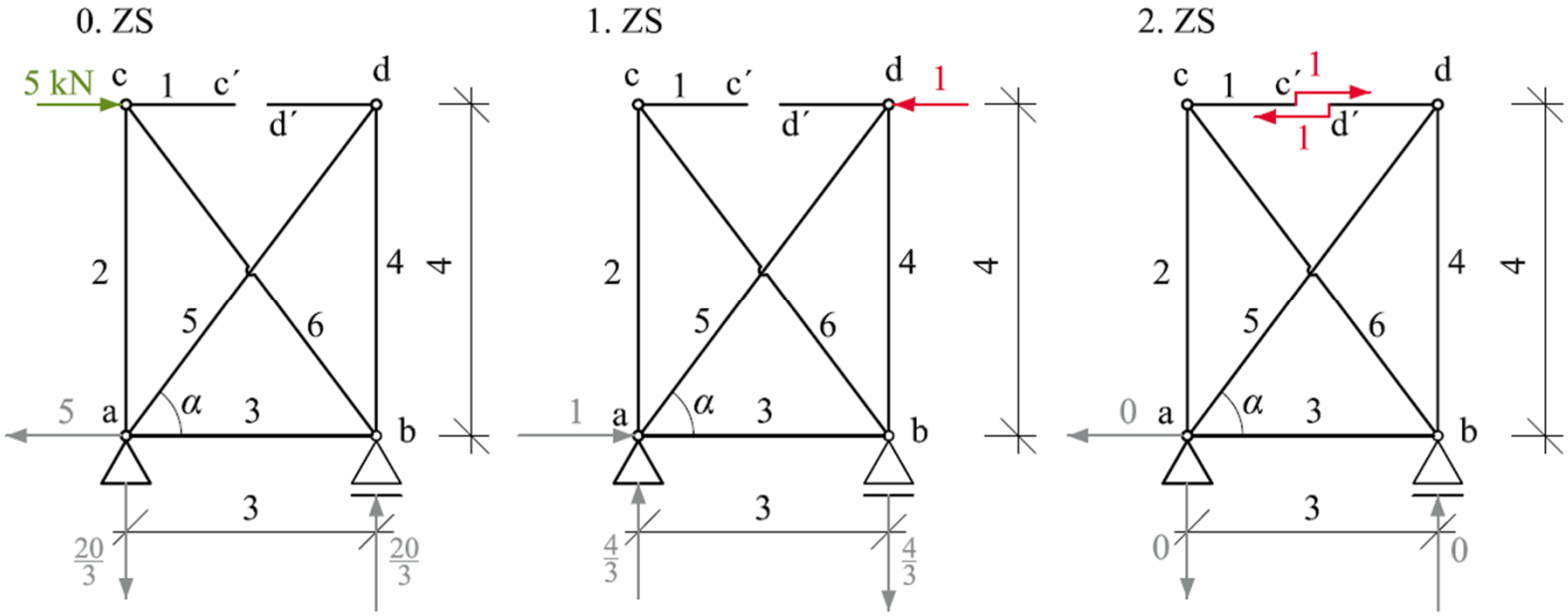
$$X_1 = 4,\overline{4} \cdot 10^3 \text{ N} = \mathbf{4,\overline{4}\ kN}$$

SILOVÁ METODA

Princip superpozice

- $N_j = N_{0j} + N_{1j} \cdot X_1 + N_{2j} \cdot X_2$
- $R_r = R_{r0} + R_{r1} \cdot X_1 + R_{r2} \cdot X_2$





SILOVÁ METODA

Osové síly

$$N_j = N_{0j} + N_{1j} \cdot X_1 + N_{2j} \cdot X_2$$

$$N_1 = 0 + 0 \cdot 4,\overline{4} + 1 \cdot (-4,\overline{4}) = -4,\overline{4}\ \text{kN}$$

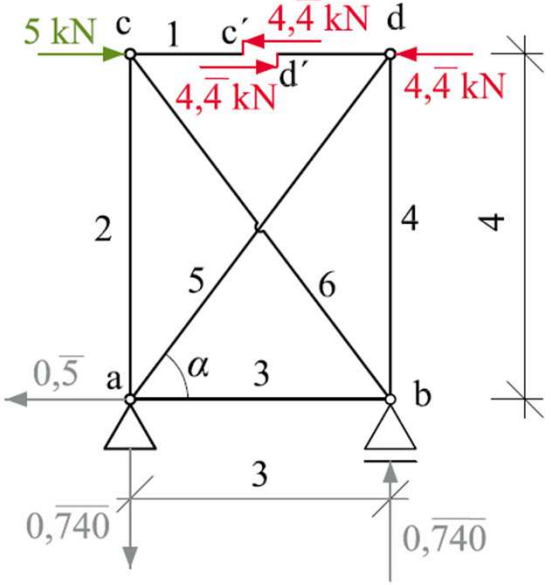
$$N_2 = \frac{20}{3} + 0 \cdot 4,\overline{4} + \frac{4}{3} \cdot (-4,\overline{4}) = 0,\overline{740}\ \text{kN}$$

$$N_3 = 5 + 0 \cdot 4,\overline{4} + 1 \cdot (-4,\overline{4}) = 0,\overline{5}\ \text{kN}$$

$$N_4 = 0 + \frac{4}{3} \cdot 4,\overline{4} + \frac{4}{3} \cdot (-4,\overline{4}) = 0$$

$$N_5 = 0 - \frac{5}{3} \cdot 4,\overline{4} - \frac{5}{3} \cdot (-4,\overline{4}) = 0$$

$$N_6 = -\frac{25}{3} + 0 \cdot 4,\overline{4} - \frac{5}{3} \cdot (-4,\overline{4}) = -0,\overline{925}\ \text{kN}$$



Reakce

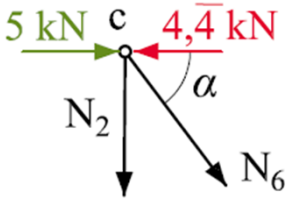
$$R_r = R_{r0} + R_{r1} \cdot X_1 + R_{r2} \cdot X_2$$

$$R_{a,x} = -5 + 1 \cdot 4,\overline{4} + 0 \cdot (-4,\overline{4}) = 0,\overline{5}\ \text{kN}$$

$$R_{a,z} = -\frac{20}{3} + \frac{4}{3} \cdot 4,\overline{4} + 0 \cdot (-4,\overline{4}) = -0,\overline{740}\ \text{kN}$$

$$R_{b,z} = \frac{20}{3} - \frac{4}{3} \cdot 4,\overline{4} + 0 \cdot (-4,\overline{4}) = 0,\overline{740}\ \text{kN}$$

SILOVÁ METODA



styčník „c"

- $\sum F_{i,x} = 0;\ 5 - 4,\overline{4} + \cos\alpha \cdot N_6 = 0 \rightarrow \boldsymbol{N_6 = -0,\overline{925}\ \mathbf{kN}}$
- $\sum F_{i,z} = 0;\ N_2 + \sin\alpha \cdot N_6 = 0 \rightarrow \boldsymbol{N_2 = 0,\overline{740}\ \mathbf{kN}}$

styčník „a"

- $\sum F_{i,z} = 0;\ 0,\overline{740} - N_2 - \sin\alpha \cdot N_5 = 0 \rightarrow \boldsymbol{N_5 = 0}$
- $\sum F_{i,x} = 0;\ -0,\overline{5} + N_3 + \cos\alpha \cdot N_5 = 0 \rightarrow \boldsymbol{N_3 = 0,\overline{5}\ \mathbf{kN}}$

styčník „b"

- $\sum F_{i,z} = 0;\ -0,\overline{740} - N_4 - \sin\alpha \cdot N_6 = 0 \rightarrow \boldsymbol{N_4 = 0}$
- $\sum F_{i,x} = 0;\ -N_3 - \cos\alpha \cdot N_6 = 0 \rightarrow 0 = 0$