VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ
FAKULTA STAVEBNÍ

BDA016 Stavební mechanika 2
6. přednáška

- Rovinný rám řešený silovou metodou

doc. Ing. Hana Šimonová, Ph.D. (hana.simonova@vut.cz)

V přednášce jsou použity obrázky z učebnice Kadlčák, J., Kytýr, J. Statika stavebních konstrukcí II. Staticky neurčité prutové konstrukce. Nakladatelství VUTIUM v Brně, 2004.

Stupeň statické neurčitosti

- $n_s = r - m = 6 - 3 = 3$

Základní soustava – SU

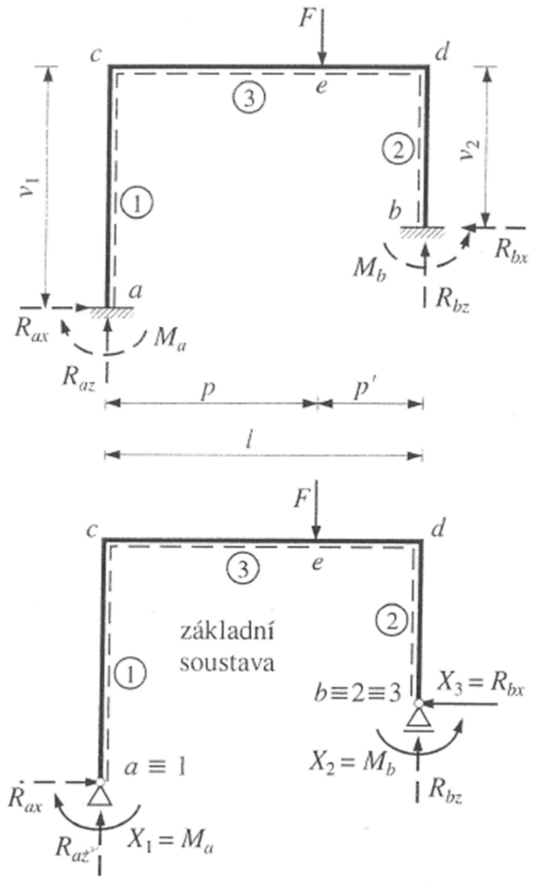
- odebrání 3 vazeb

Staticky neurčité veličiny

- 3 složky reakcí
- $X_1 = M_a$
- $X_2 = M_b$
- $X_3 = R_{bx}$

Deformační podmínky

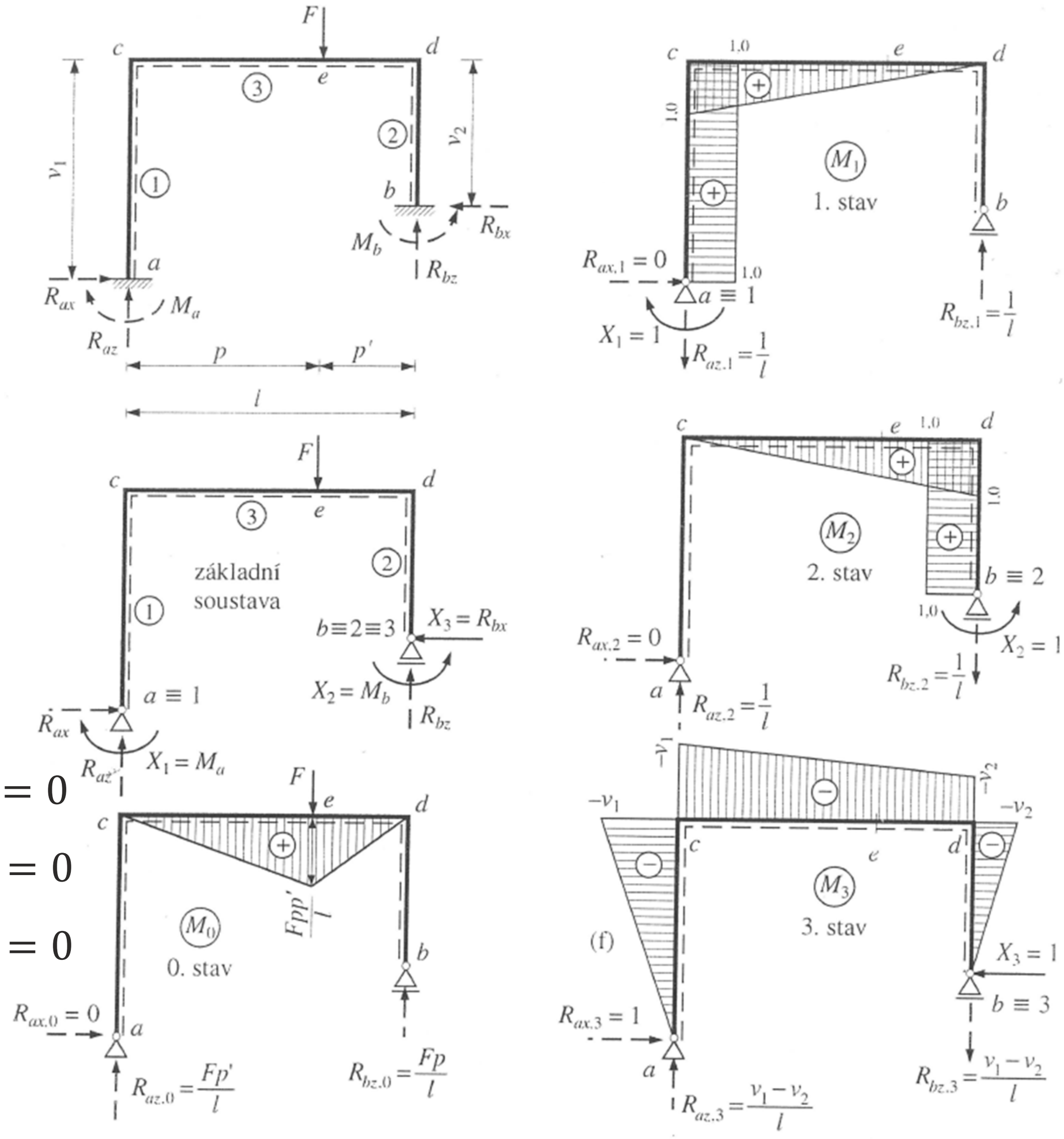
- $\delta_1 = \varphi_a = 0$
- $\delta_2 = \varphi_b = 0$
- $\delta_3 = u_b = 0$



SILOVÁ METODA – ROVINNÝ RÁM

Princip superpozice
→ 4 zatěžovací stavy

- 0\. ZS – zatížení
- 1\. ZS – $X_1 = M_a = 1$
- 2\. ZS – $X_2 = M_b = 1$
- 3\. ZS – $X_3 = R_{bx} = 1$



Systém lineárních rovnic

- $\delta_1 = \delta_{10} + \delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 + \delta_{13} \cdot X_3 = 0$
- $\delta_2 = \delta_{20} + \delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 + \delta_{23} \cdot X_3 = 0$
- $\delta_3 = \delta_{30} + \delta_{31} \cdot X_1 + \delta_{32} \cdot X_2 + \delta_{33} \cdot X_3 = 0$

kanonické rovnice silové metody

- $\sum_{k=1}^{n_s} \delta_{ik} \cdot X_k + \delta_{i0} = 0$

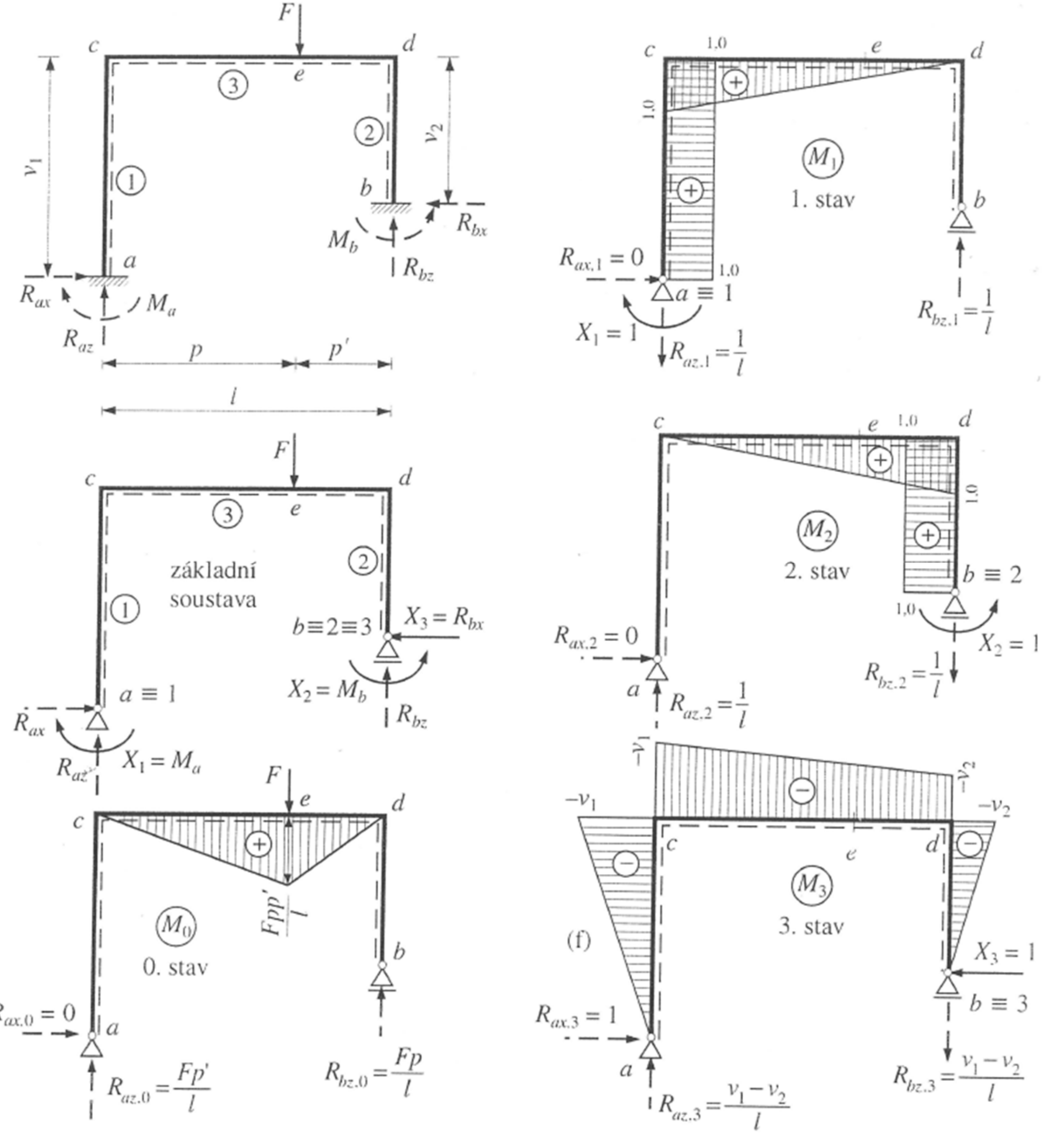
SILOVÁ METODA – ROVINNÝ RÁM

Deformační součinitele

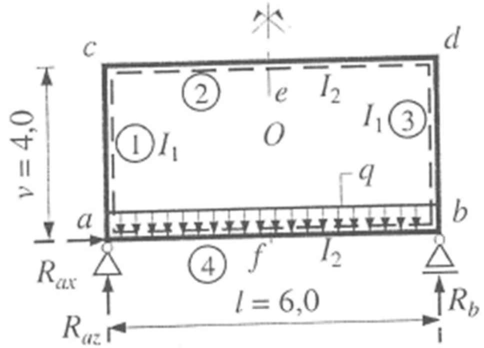
→ z principu virtuálních prací

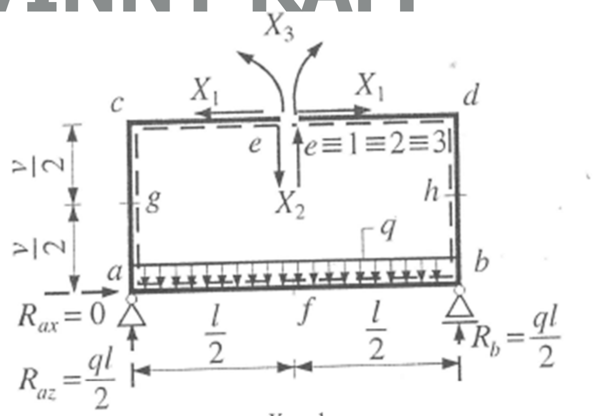
pro silové zatížení

- $\delta_{i0} = \int_0^s \frac{N_i N_0}{EA} \mathrm{d}s + \int_0^s \varkappa \frac{V_i V_0}{GA} \mathrm{d}s +$
  $+ \int_0^s \boldsymbol{\frac{M_i M_0}{EI} ds}$
- $\delta_{ik} = \int_0^s \frac{N_i N_k}{EA} \mathrm{d}s + \int_0^s \varkappa \frac{V_i V_k}{GA} \mathrm{d}s +$
  $+ \int_0^s \boldsymbol{\frac{M_i M_k}{EI} ds}$
- $i = 1, 2, \cdots, n_s$
- $\delta_{ik} = \delta_{ki}$



SILOVÁ METODA – ROVINNÝ RÁM



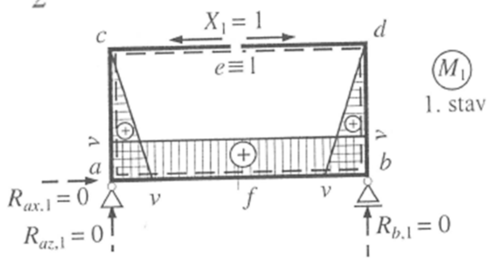


Stupeň statické neurčitosti

- $n_s = r - m = 6 - 3 = 3$
- $n_s = 3 \cdot 1 - 0 + (3 - 3) = 3$

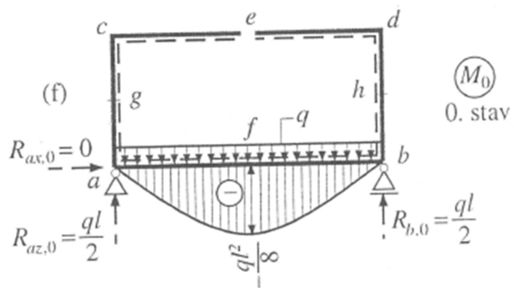
Základní soustava – SU

- odebrání 3 vazeb – vnitřní

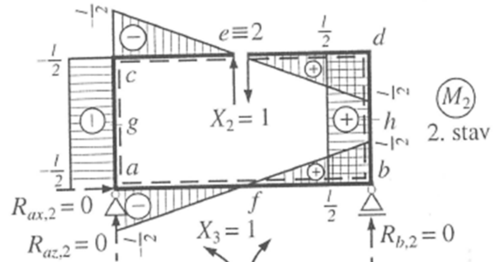


Staticky neurčité veličiny

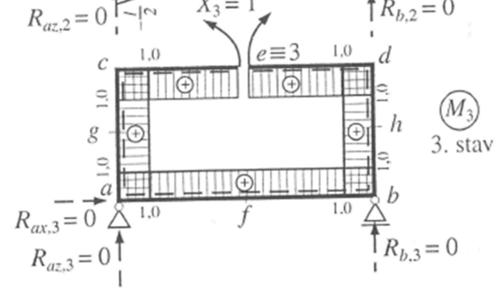
- 3 složky výslednice vnitřních sil



- $X_1 = N_e$
- $X_2 = V_e$
- $X_3 = M_e$



Deformační podmínky



- $\delta_1 = u_{ee} = 0\text{: } \delta_{10} + \delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 + \delta_{13} \cdot X_3 = 0$
- $\delta_2 = w_{ee} = 0\text{: } \delta_{20} + \delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 + \delta_{23} \cdot X_3 = 0$
- $\delta_3 = \varphi_{ee} = 0\text{: } \delta_{30} + \delta_{31} \cdot X_1 + \delta_{32} \cdot X_2 + \delta_{33} \cdot X_3 = 0$

Deformační součinitele

→ z principu virtuálních prací

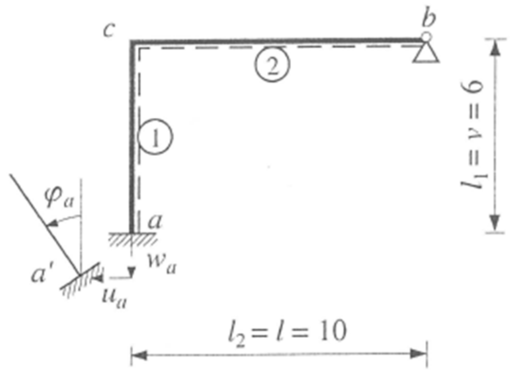
pro teplotní zatížení

- $\delta_{i0,t} = \int_0^s N_i \alpha_t \, \Delta T_0 \mathrm{d}s + \int_0^s M_i \alpha_t \frac{\Delta T}{h} \mathrm{d}s$

pro popuštění podpor

- popuštění je zadané v místě odebrané vazby
  → popuštění se objeví na pravé straně rovnice
- popuštění je zadané v místě ponechané vazby
  → popuštění se uvažuje v 0.ZS
  $\delta_{i0,p} = -\sum_{r=1}^{p_v} R_{r,i} \cdot \delta_r =$

  $= -\sum_{r=1}^{p_v} \left(R_{rx,i} \cdot u_r + R_{rz,i} \cdot w_r + R_{rM,i} \cdot \varphi_r \right)$



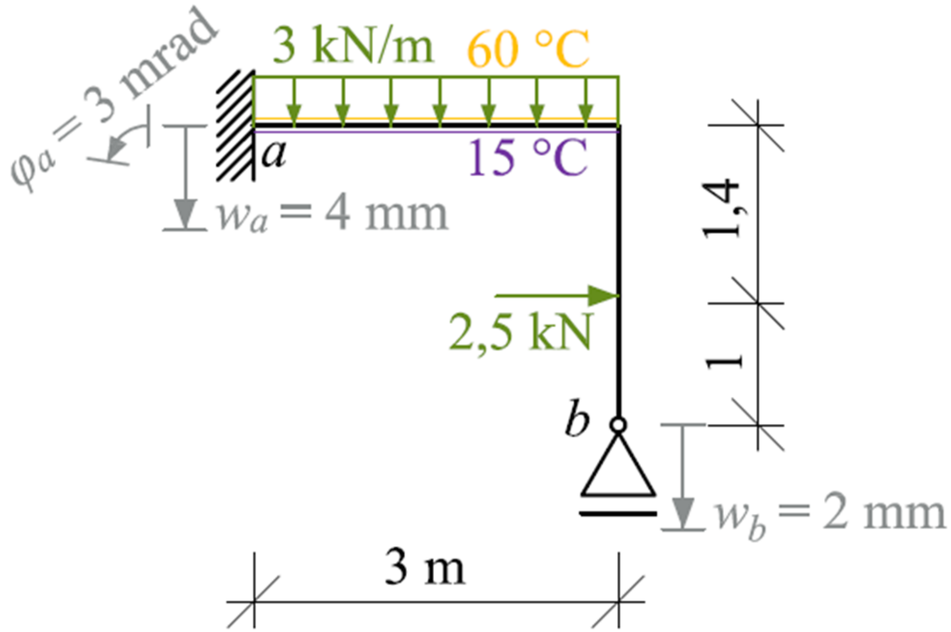
Na dané konstrukci pomocí silové metody vykreslete průběhy vnitřních sil.

$E = 210\text{GPa}$

$I = 2 \cdot 10^{-5}\ \text{m}^4$

$h = 0{,}2\ \text{m}$

$\alpha_t = 1{,}2 \cdot 10^{-5}\ °\text{C}^{-1}$



$EI = 4{,}2 \cdot 10^6 \text{Nm}^2$

$\Delta T_d = 15\ °\text{C}$

$\Delta T_h = 60\ °\text{C}$

$$\Delta T_0 = \frac{\Delta T_d + \Delta T_h}{2} = \frac{15 + 60}{2} = 37{,}5\ °\text{C}$$

$$\Delta T = \Delta T_d - \Delta T_h = 15 - 60 = -45\ °\text{C}$$

Stupeň statické neurčitosti

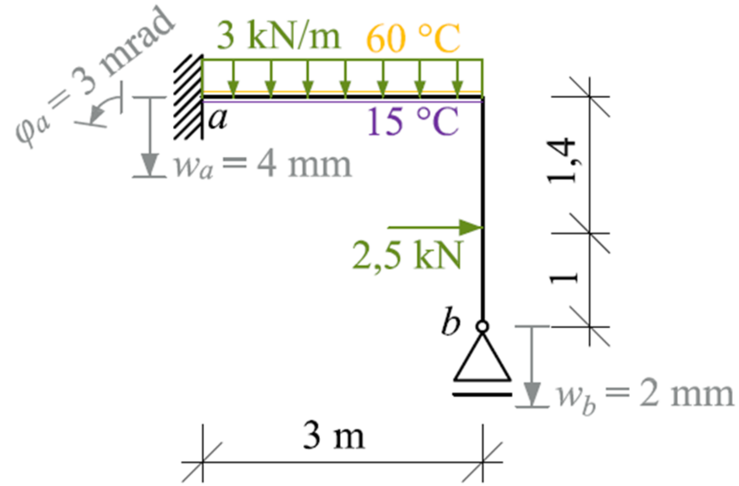
- $n_s = 4 - 3 = 1$

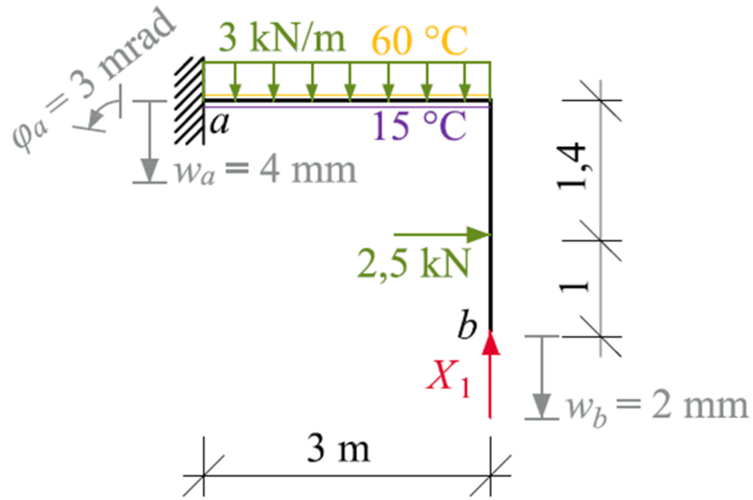
Základní soustava – staticky určitá

- odebereme n_s přebytečných vazeb
- nahradíme staticky neurčitými veličinami

Deformační podmínka

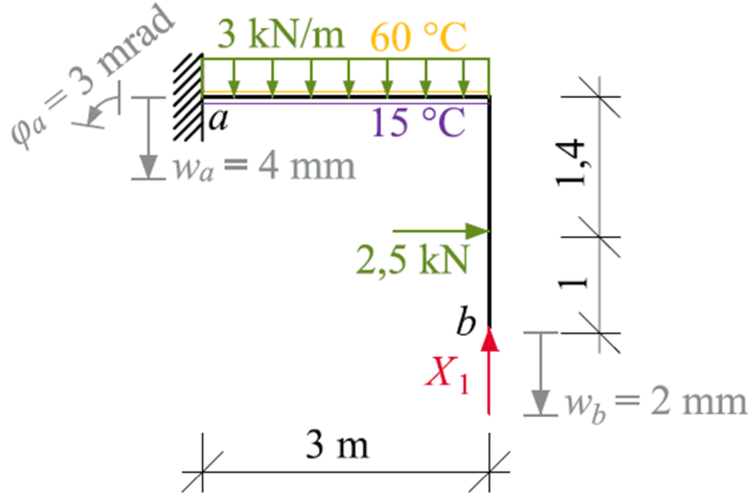
- $w_b = \delta_1 = -0{,}002 \text{ m}$
- $w_b = w_{b,z} + w_{b,X_1}$
- $\boldsymbol{\delta_{10} + \delta_{11} \cdot X_1 = -0,002}$

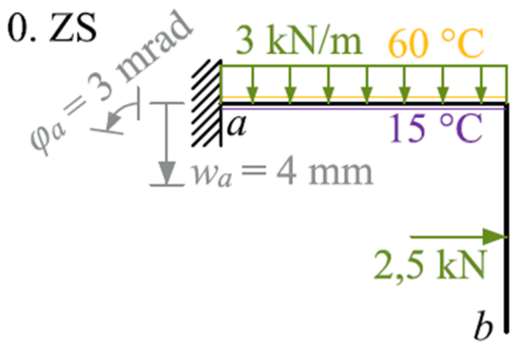


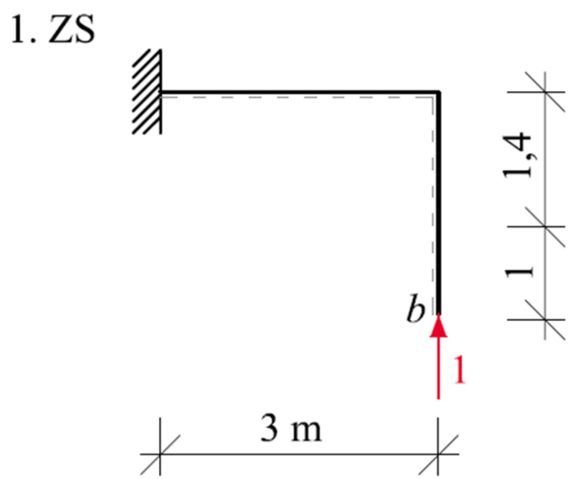


Zatěžovací stavy – princip superpozice

- $\boldsymbol{\delta_{10} + \delta_{11} \cdot X_1 = -0,002}$
- $\delta_{10} = \delta_{10,sz} + \delta_{10,t} + \delta_{10,p}$
- $\delta_{11} = \int_0^s \frac{M_1 \cdot M_1}{EI} \mathrm{d}s$

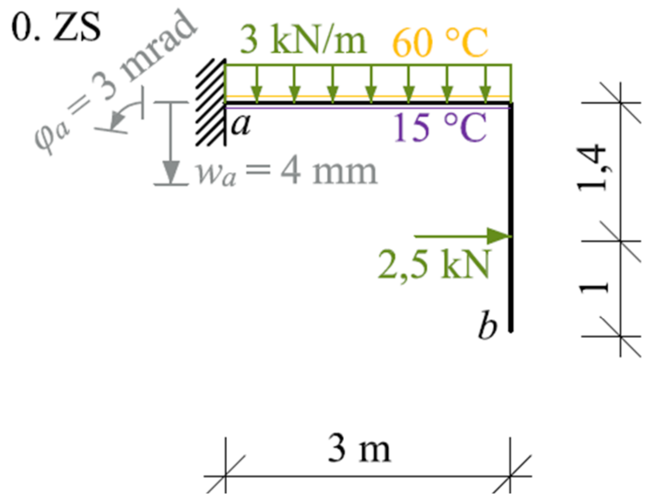




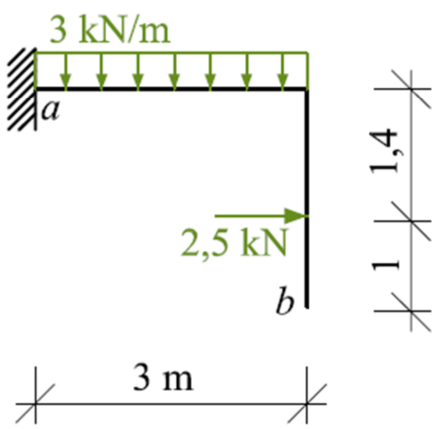


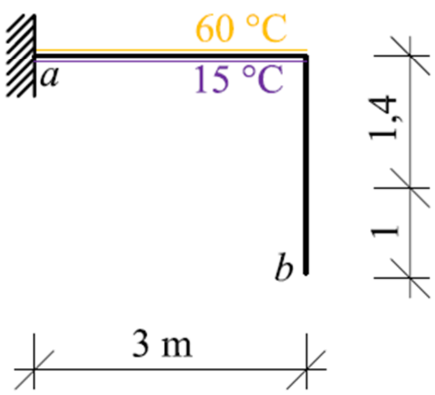
0.ZS

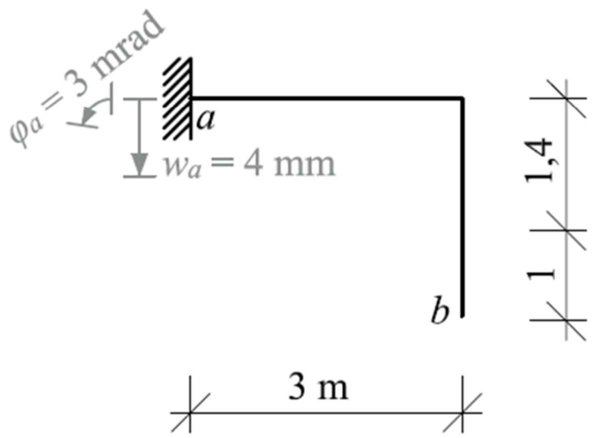
- $\delta_{10} = \delta_{10,sz} + \delta_{10,t} + \delta_{10,p}$
- $\delta_{10,sz} = \int_0^s \frac{M_1 \cdot M_0}{EI} \mathrm{d}s$
- $\delta_{10,t} = \int_0^s N_1 \alpha_t \, \Delta T_0 \mathrm{d}s + \int_0^s M_1 \alpha_t \frac{\Delta T}{h} \mathrm{d}s$
- $\delta_{10,p} = -\sum_{r=1}^{p_v} R_{r,1} \cdot \delta_r$



0. ZS

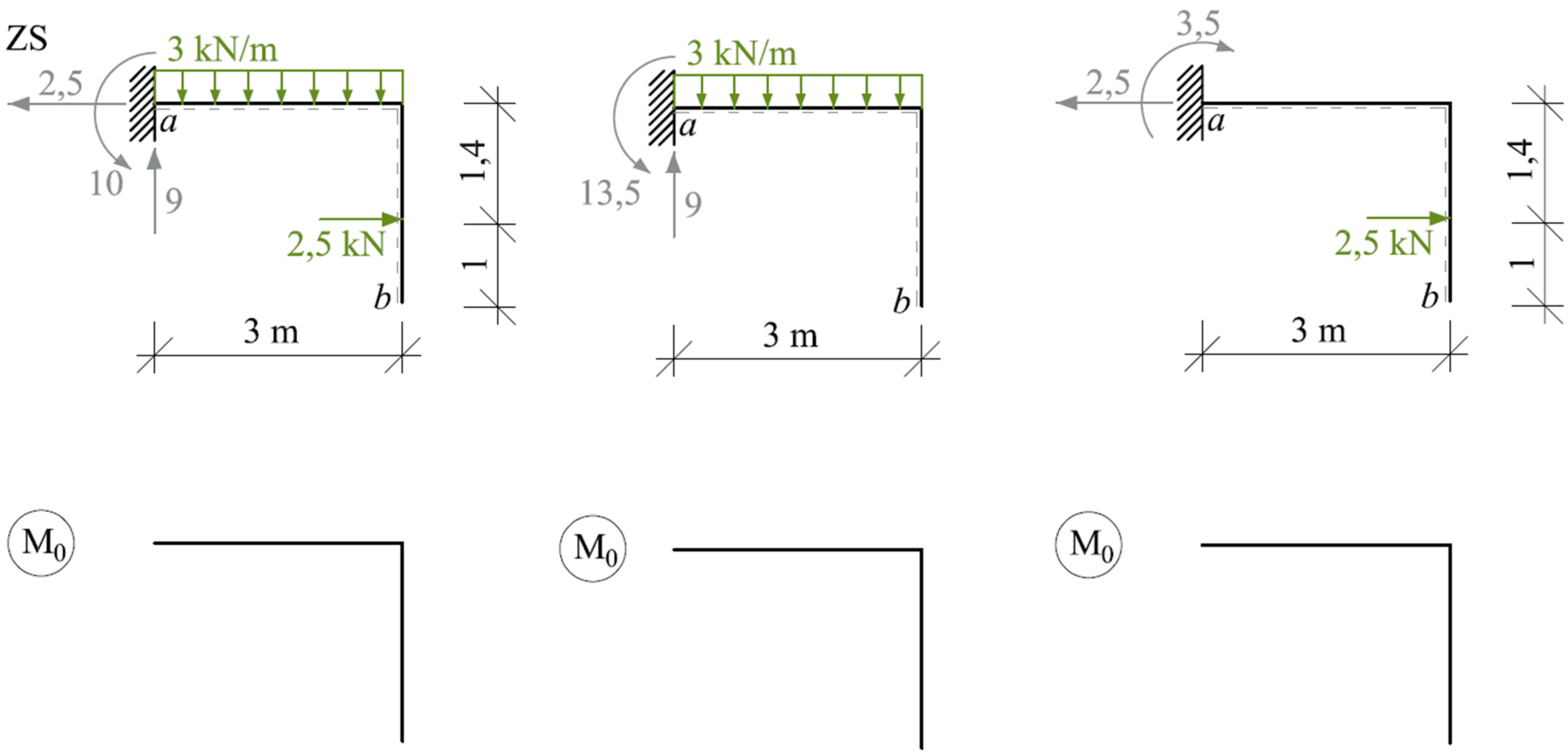






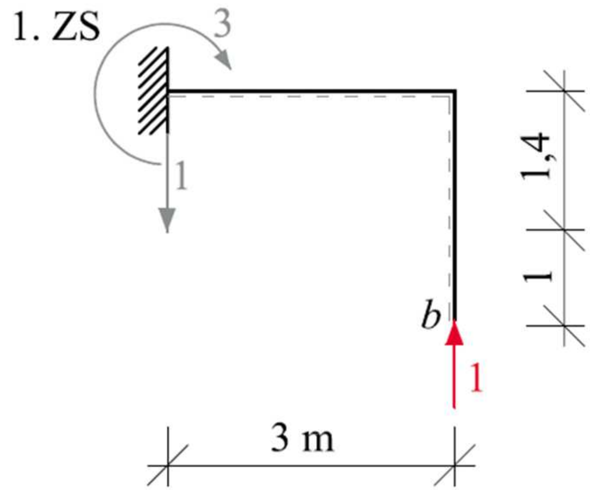
SILOVÁ METODA – ROVINNÝ RÁM

0. ZS



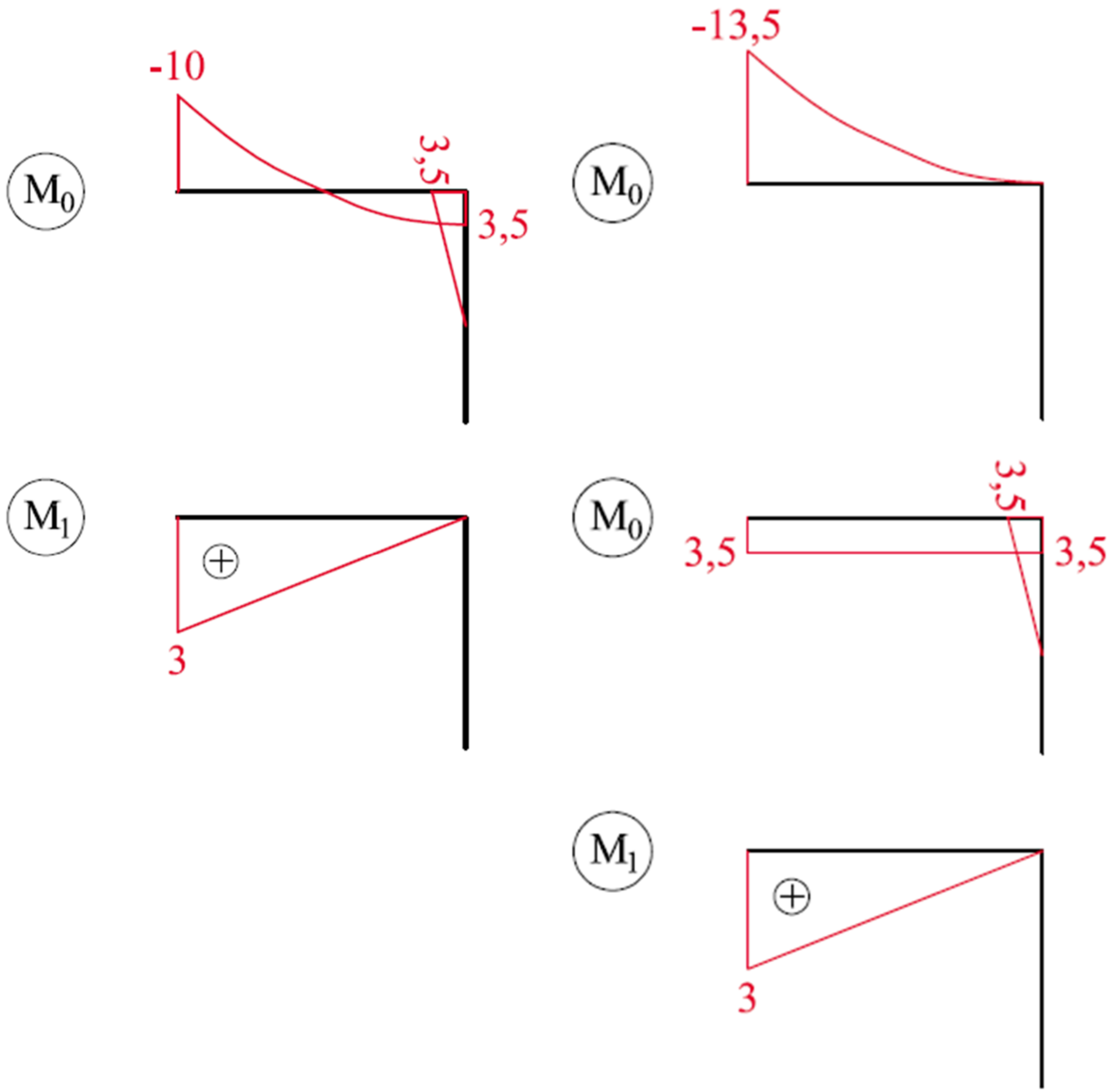
1.ZS

$$\delta_{11} = \int_0^s \frac{M_1 \cdot M_1}{EI} \mathrm{d}s$$



SILOVÁ METODA – ROVINNÝ RÁM

$$\delta_{10,sz} = \int_0^s \frac{M_1 \cdot M_0}{EI} \mathrm{d}s =$$

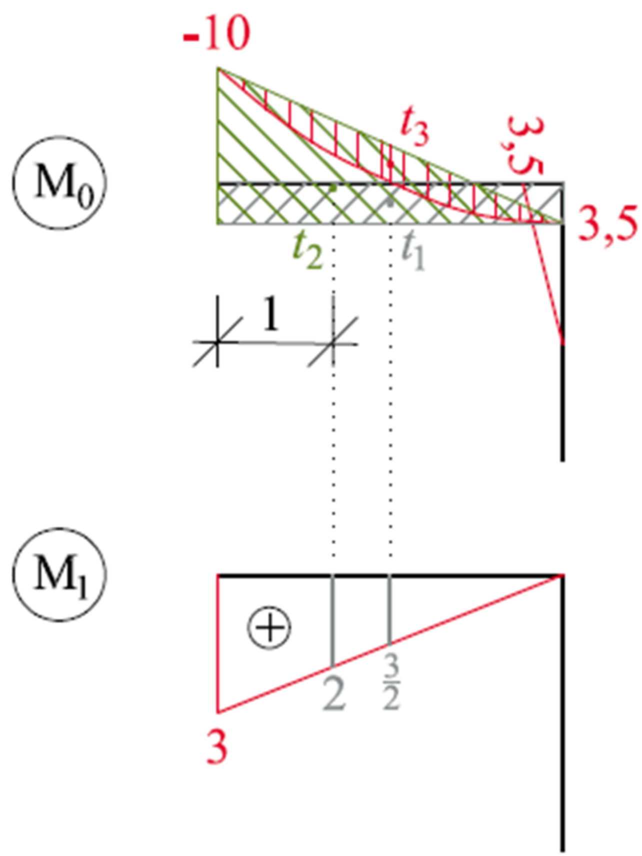


$$\delta_{10,sz} = \int_0^S \frac{M_1 \cdot M_0}{EI} \mathrm{d}s = \frac{1}{EI} \int_0^S M_1 \cdot M_0 \mathrm{d}s =$$

$$= \frac{1 \cdot 10^3}{4{,}2 \cdot 10^6} \left(3 \cdot 3{,}5 \cdot \frac{3}{2} + \frac{1}{2} \cdot 3 \cdot (-13{,}5) \cdot 2 + \frac{2}{3} \cdot 3 \cdot 3{,}375 \cdot \frac{3}{2} \right) =$$

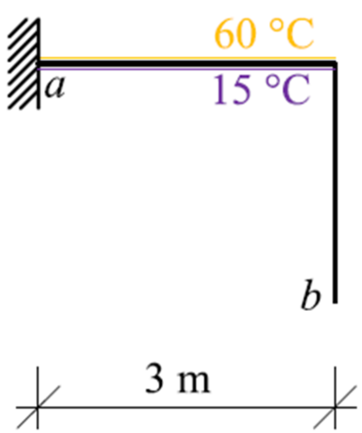
$$= \frac{-14{,}625 \cdot 10^3}{4{,}2 \cdot 10^6} \rightarrow$$

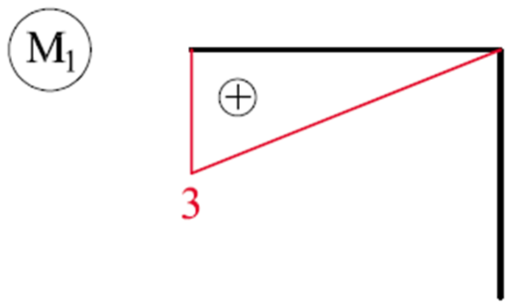
$$\boldsymbol{\delta_{10,sz} = -3{,}482 \cdot 10^{-3}\ \mathrm{m}}$$



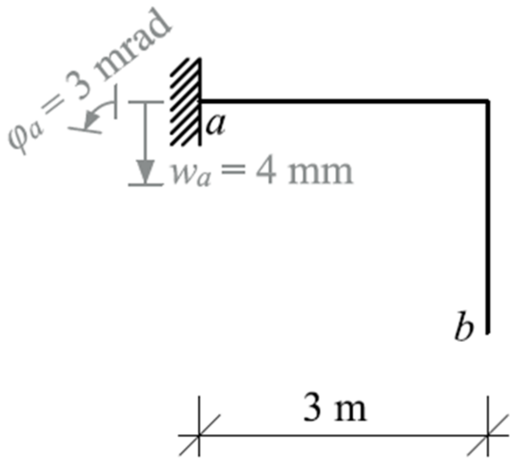
SILOVÁ METODA – ROVINNÝ RÁM

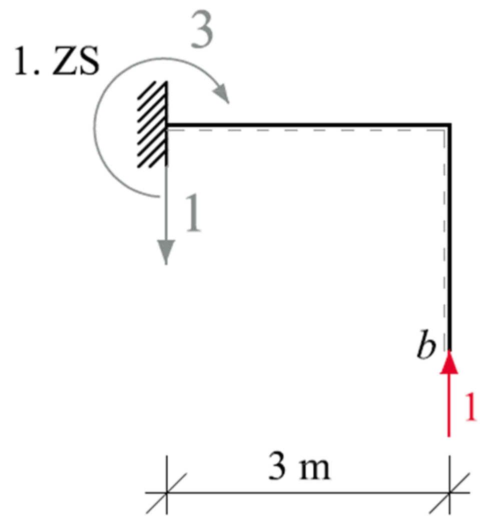
$$\delta_{10,t} = \int_0^s N_1 \alpha_t \, \Delta T_0 \mathrm{d}s + \int_0^s M_1 \alpha_t \frac{\Delta T}{h} \mathrm{d}s$$





$$\delta_{10,p} = -\sum_{r=1}^{p_v} R_{r,1} \cdot \delta_r$$





SILOVÁ METODA – ROVINNÝ RÁM

$$\boldsymbol{\delta_{10} + \delta_{11} \cdot X_1 = -0,002}$$

$$\delta_{10} = \delta_{10,sz} + \delta_{10,t} + \delta_{10,p} =$$

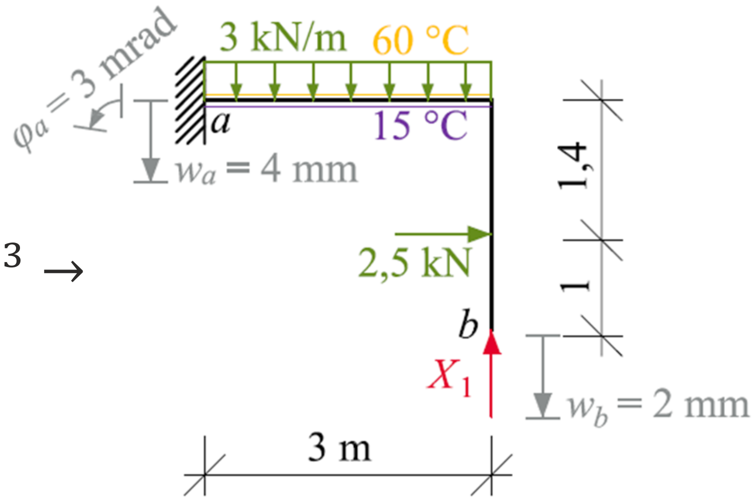
$$= -3{,}482 \cdot 10^{-3} + (-12{,}15 \cdot 10^{-3}) + 5 \cdot 10^{-3} \rightarrow$$

$$\boldsymbol{\delta_{10} = -10,632 \cdot 10^{-3}\ \mathrm{m}}$$

$$\boldsymbol{\delta_{11} = 2,143 \cdot 10^{-6}\ \frac{\mathrm{m}}{\mathrm{N}}}$$

$$\boldsymbol{-10,632 \cdot 10^{-3} + 2,143 \cdot 10^{-6} \cdot X_1 = -0,002 \rightarrow}$$

$$\boldsymbol{X_1 = 4\,028\ \mathrm{N} = 4,028\ \mathrm{kN}}$$



SILOVÁ METODA – ROVINNÝ RÁM