VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ
FAKULTA STAVEBNÍ

BDA015 Stavební mechanika 1
5. přednáška

- Řešení rovinných přímých nosníků
- Nosník prostý a konzolový
- Nosník s převislými konci

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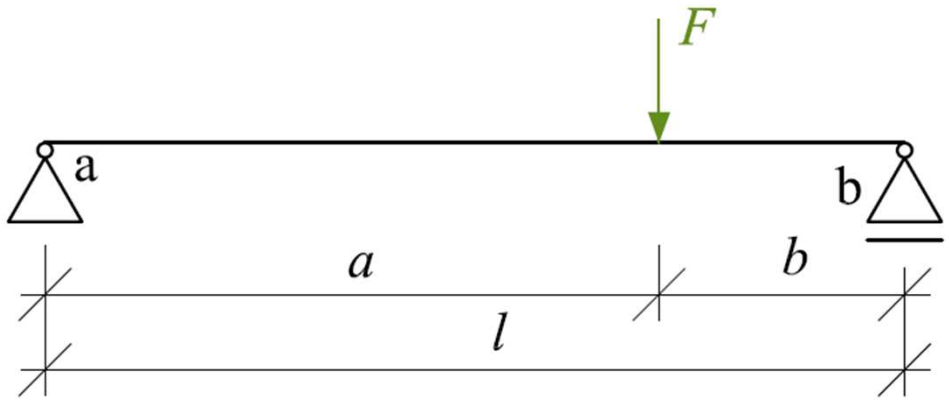
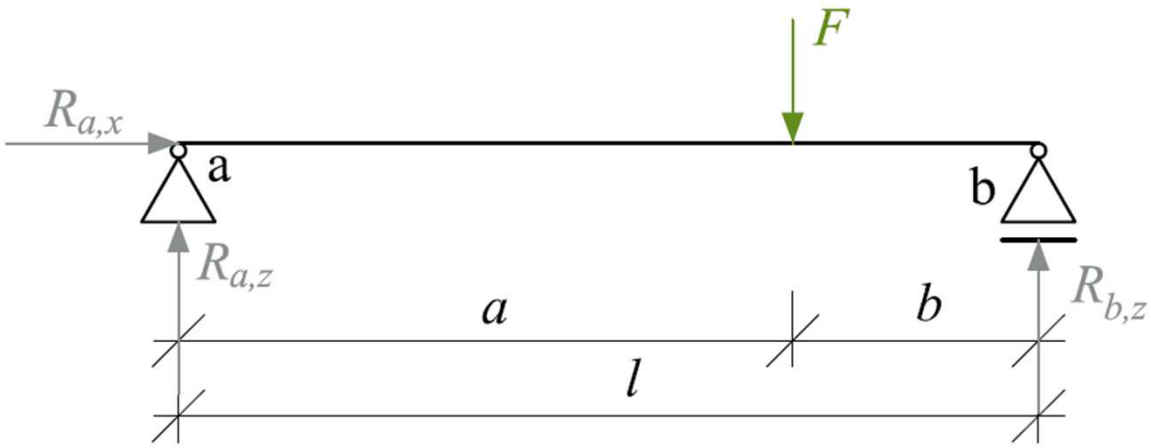
ROVINNÝ PROSTÝ NOSNÍK

Vykreslete průběhy vnitřních sil



ROVINNÝ PROSTÝ NOSNÍK

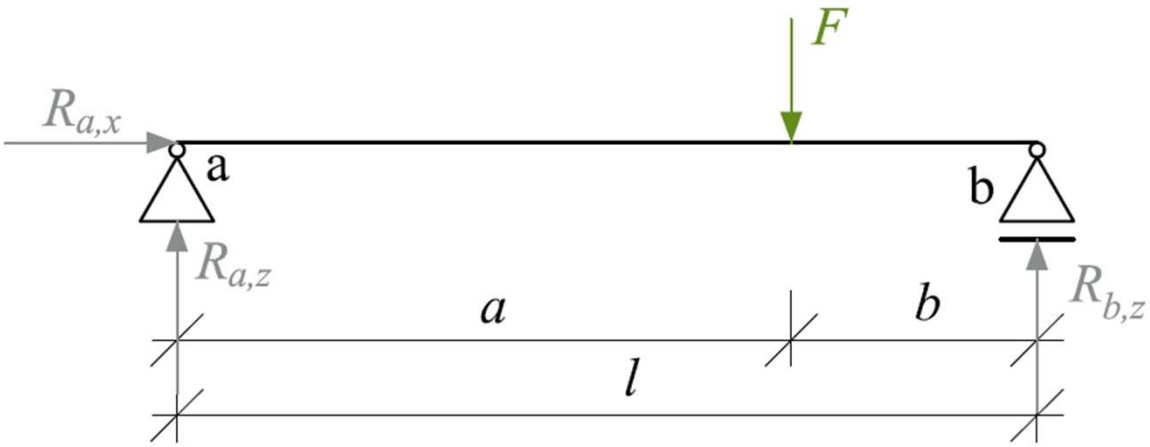
1) Výpočet reakcí

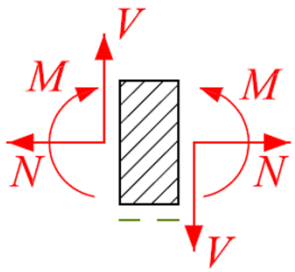


- $\sum F_{i,x} = 0 \rightarrow R_{a,x} = 0$ ⊕ →
- $\sum M_{i,a} = 0; \quad -F \cdot a + R_{b,z} \cdot l = 0 \rightarrow R_{b,z} = \frac{F \cdot a}{l}$ ⊕
- $\sum M_{i,b} = 0; \; -R_{a,z} \cdot l + F \cdot b = 0 \rightarrow R_{a,z} = \frac{F \cdot b}{l}$
- $\sum F_{i,z} = 0; \; F - R_{a,z} - R_{b,z} = 0 \,; 0 = 0 \rightarrow$ VYHOVÍ ⊕ ↓

ROVINNÝ PROSTÝ NOSNÍK

2) Posouvající síly

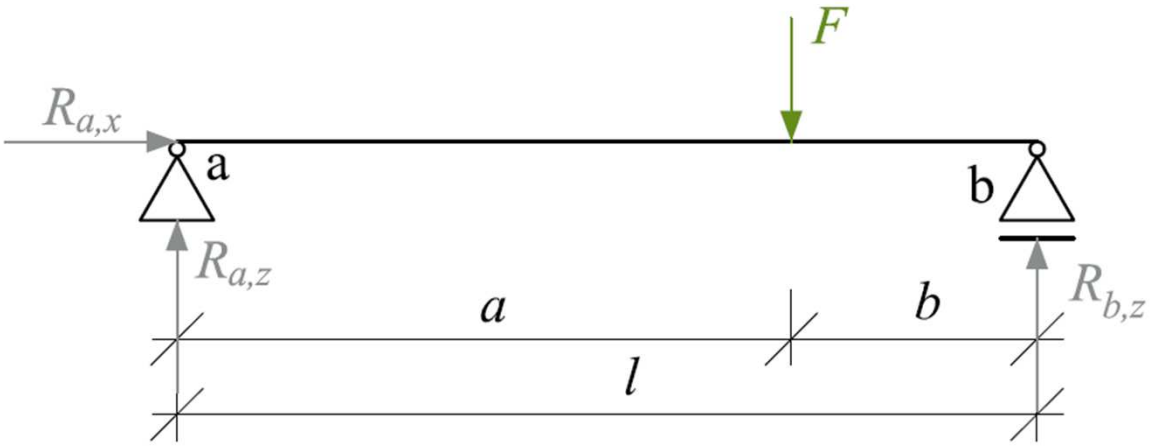


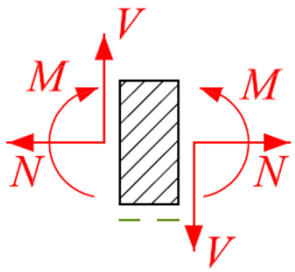


- $V_a^L = R_{a,z} = \frac{F \cdot b}{l}$
- $V_{Fa}^L = R_{a,z}$ (před silou F); $V_{Fb}^L = R_{a,z} - F = \frac{F \cdot b}{l} - F = -\frac{F \cdot a}{l}$ (za silou F)
- $V_{Fb}^P = -R_{b,z}$ (před silou F); $V_{Fa}^P = -R_{b,z} + F = -\frac{F \cdot a}{l} + F = \frac{F \cdot b}{l}$ (za silou F)
- $V_b^P = -R_{b,z} = -\frac{F \cdot a}{l}$

ROVINNÝ PROSTÝ NOSNÍK

3) Ohybové momenty

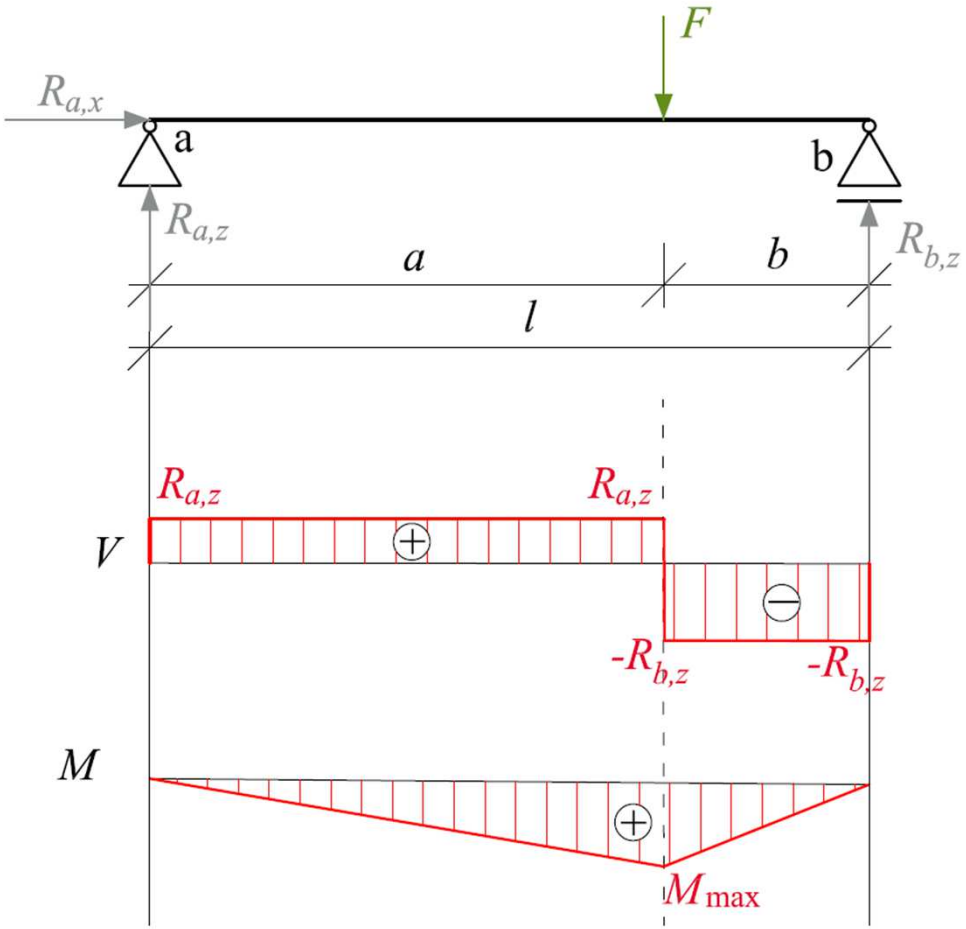


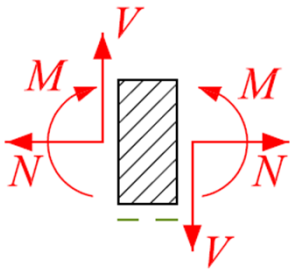


- $M_a^L = 0$
- $M_F^L = R_{a,z} \cdot a = \frac{F \cdot b}{l} \cdot a = \frac{F \cdot a \cdot b}{l} = M_{\max}$
- $M_F^P = R_{b,z} \cdot b = \frac{F \cdot a}{l} \cdot b = \frac{F \cdot a \cdot b}{l} = M_{\max}$
- $M_b^P = 0$

ROVINNÝ PROSTÝ NOSNÍK

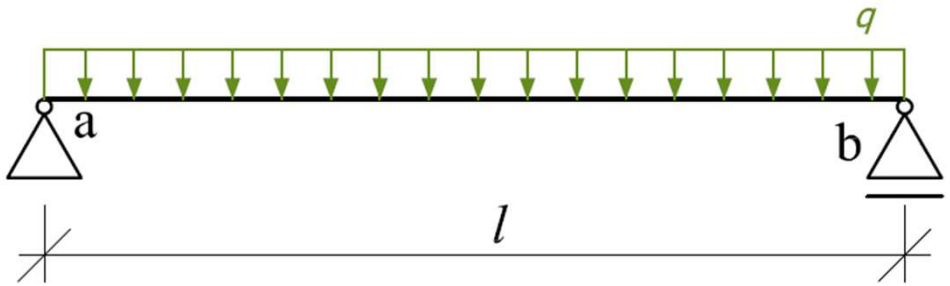
4) Průběhy vnitřních sil





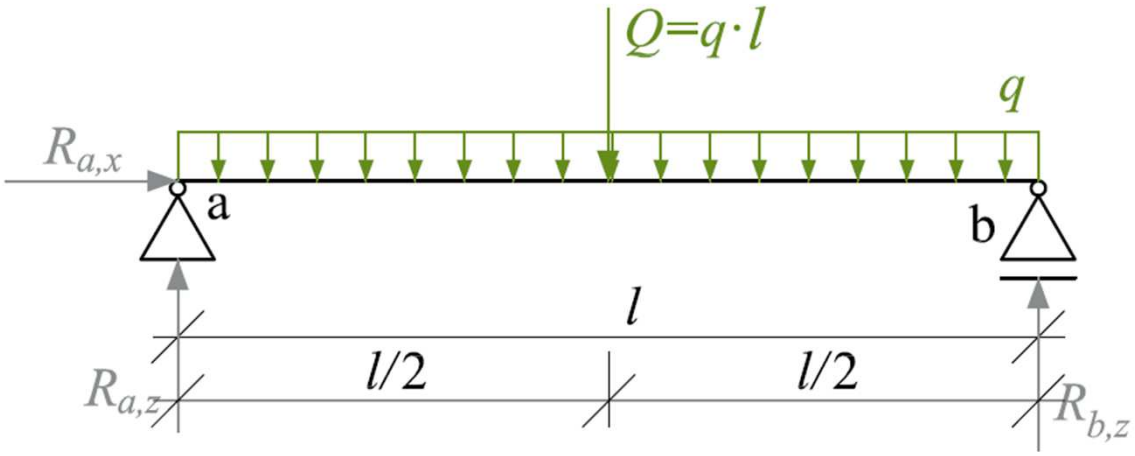
ROVINNÝ PROSTÝ NOSNÍK

Vykreslete průběhy vnitřních sil



ROVINNÝ PROSTÝ NOSNÍK

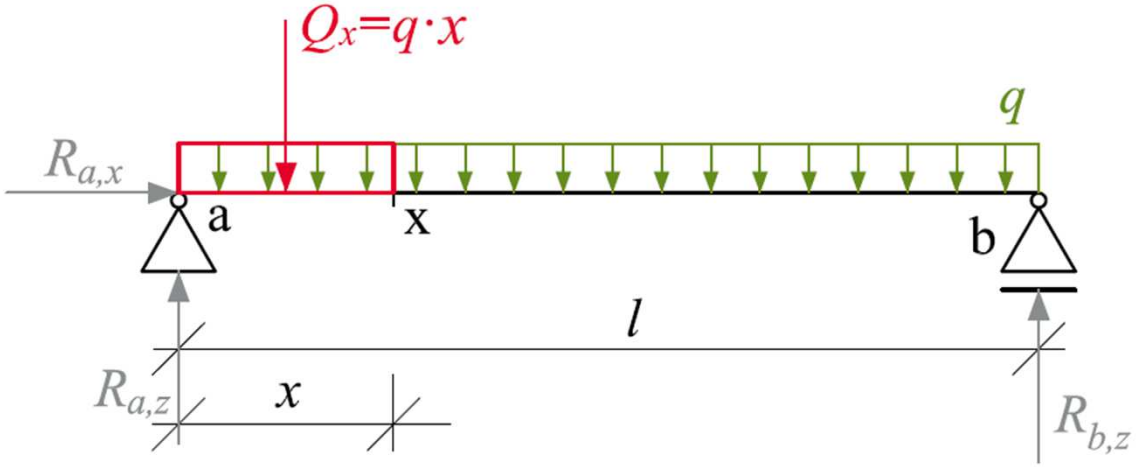
1) Výpočet reakcí

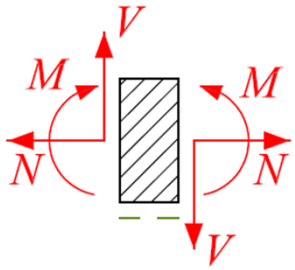


- $\sum F_{i,x} = 0 \rightarrow R_{a,x} = 0$ ⊕ →
- $\sum M_{i,a} = 0; \quad -Q \cdot \frac{l}{2} + R_{b,z} \cdot l = 0 \rightarrow R_{b,z} = \frac{Q}{2} = \frac{q \cdot l}{2}$ ⊕
- $\sum M_{i,b} = 0; \; -R_{a,z} \cdot l + Q \cdot \frac{l}{2} = 0 \rightarrow R_{a,z} = \frac{Q}{2} = \frac{q \cdot l}{2}$

ROVINNÝ PROSTÝ NOSNÍK

2) Posouvající síly

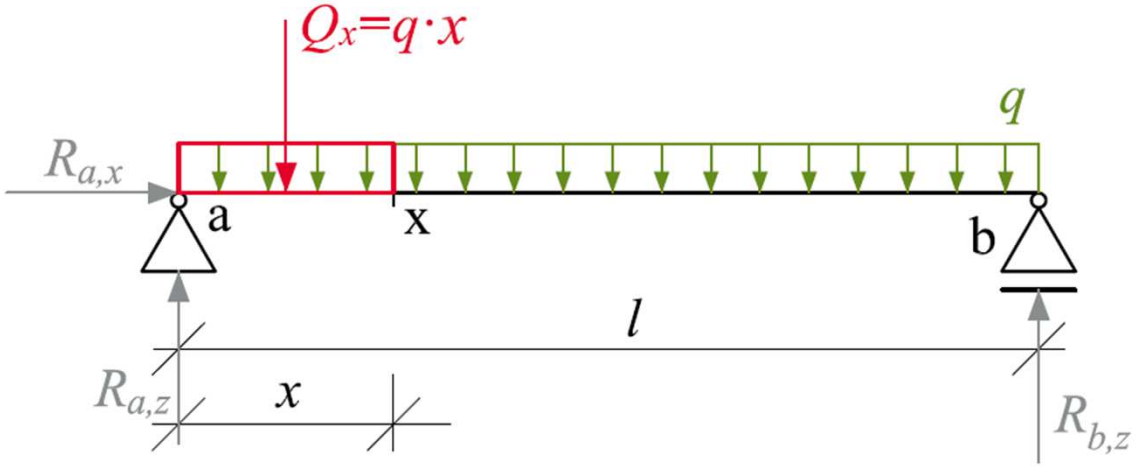


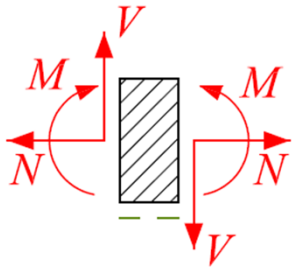


- $V_a^L = R_{a,z} = \frac{q\cdot l}{2}$
- $V_x^L = R_{a,z} - q \cdot x$
- $V_b^P = -R_{b,z} = -\frac{q\cdot l}{2}$

ROVINNÝ PROSTÝ NOSNÍK

3) Ohybové momenty

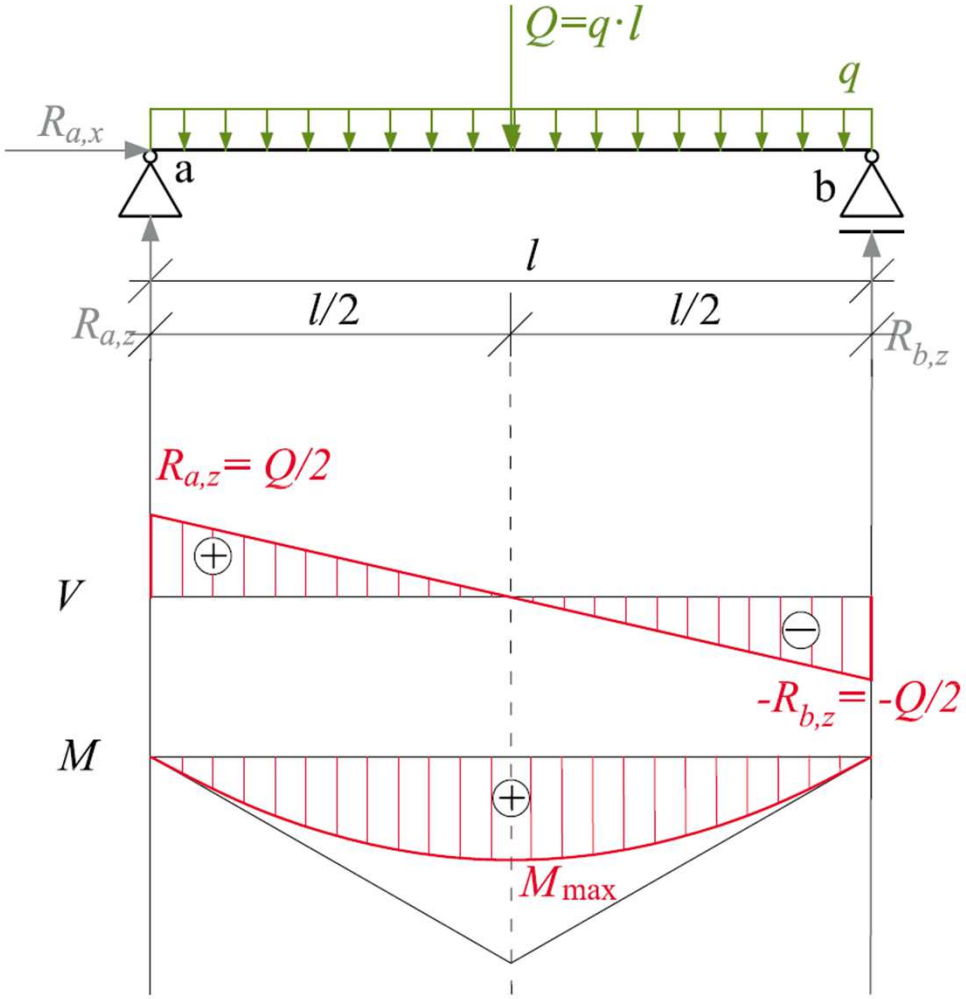


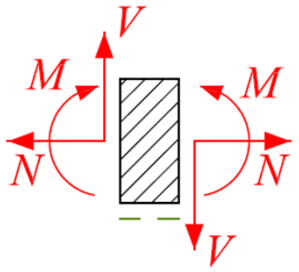


- $M_a^L = 0;\ M_b^P = 0$
- $M_x^L = R_{a,z} \cdot x - q \cdot x \cdot \frac{x}{2} = \frac{q \cdot l}{2} \cdot x - q \cdot \frac{x^2}{2}$
- $M_{l/2}^L = R_{a,z} \cdot \frac{l}{2} - q \cdot \frac{\left(\frac{l}{2}\right)^2}{2} = \frac{q \cdot l^2}{4} - \frac{q \cdot l^2}{8} = \frac{1}{8} \cdot q \cdot l^2 = M_{max}$

ROVINNÝ PROSTÝ NOSNÍK

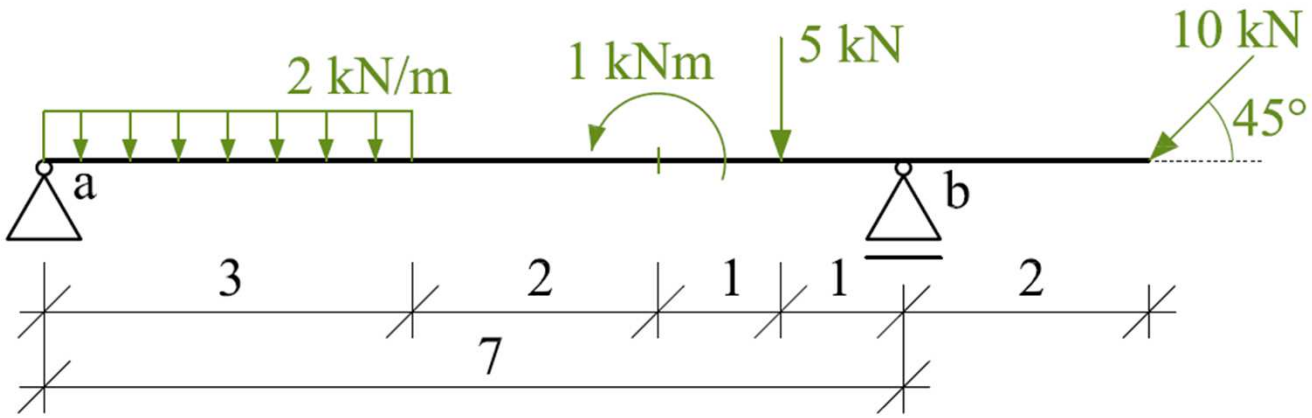
4) Průběhy vnitřních sil





PROSTÝ NOSNÍK S PŘEVISLÝM KONCEM

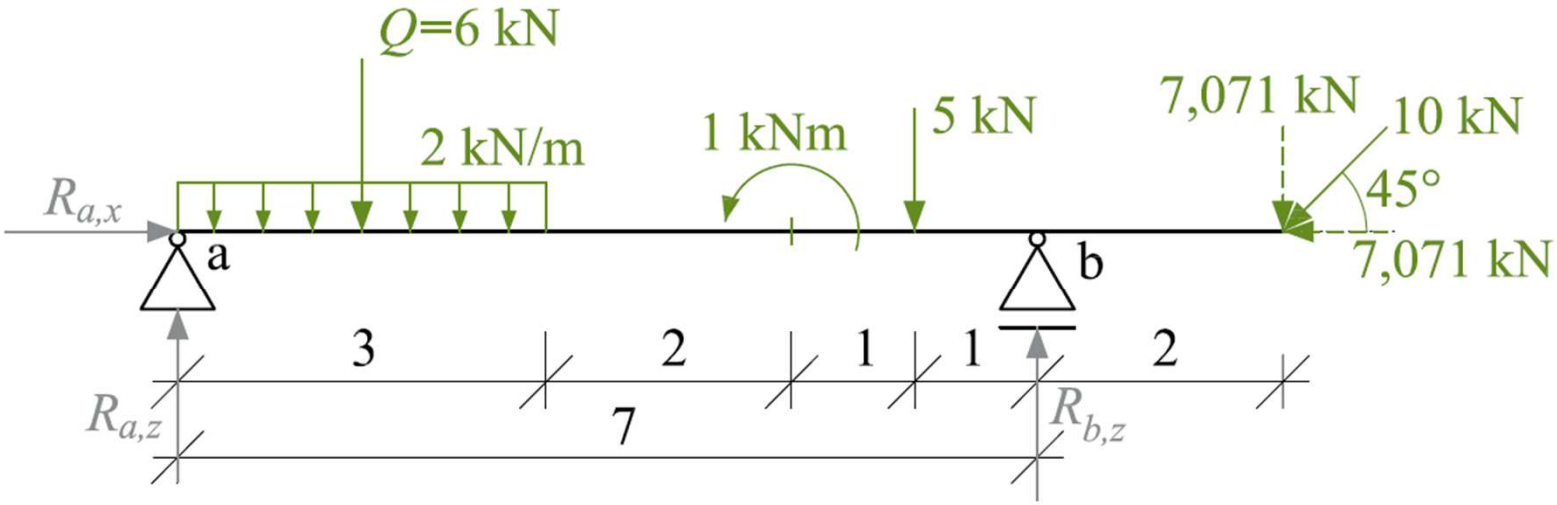
Vykreslete průběhy vnitřních sil



- $F_x = F \cdot \cos 45° = 10 \cdot \cos 45° = 7{,}071\ \text{kN}$
- $F_z = F \cdot \sin 45° = 10 \cdot \sin 45° = 7{,}071\ \text{kN}$

PROSTÝ NOSNÍK S PŘEVISLÝM KONCEM

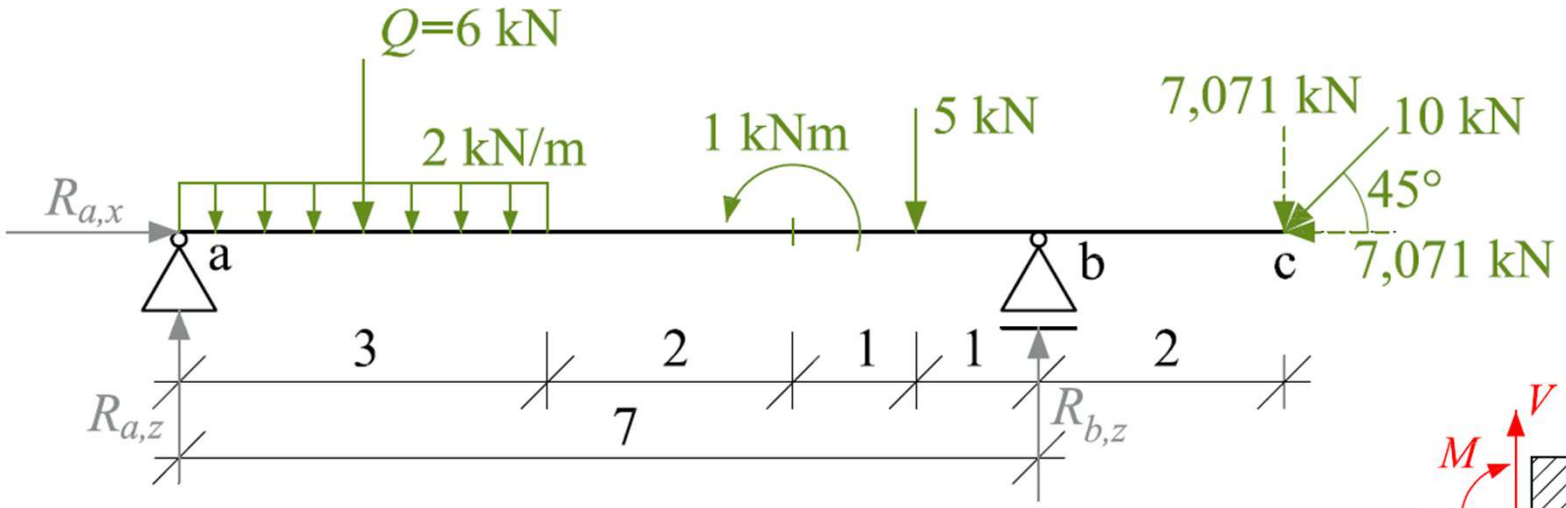
1) Výpočet reakcí

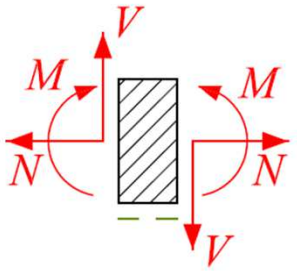


- $\sum F_{i,x} = 0 \rightarrow R_{a,x} - 7{,}071 = 0 \rightarrow R_{a,x} = 7{,}071\ \text{kN}$ ⊕ →
- $\sum M_{i,a} = 0;\ -6 \cdot 1{,}5 + 1 - 5 \cdot 6 + R_{b,z} \cdot 7 - 7{,}071 \cdot 9 = 0 \rightarrow R_{b,z} = 14{,}52\ \text{kN}$ ⊕
- $\sum M_{i,b} = 0;\ -R_{a,z} \cdot 7 + 6 \cdot 5{,}5 + 1 + 5 \cdot 1 - 7{,}071 \cdot 2 = 0 \rightarrow R_{a,z} = 3{,}551\ \text{kN}$
- $\sum F_{i,z} = 0;\ 6 + 5 + 7{,}071 - 14{,}52 - 3{,}551 = 0\,;0 = 0 \rightarrow$ VYHOVÍ ⊕ ↓

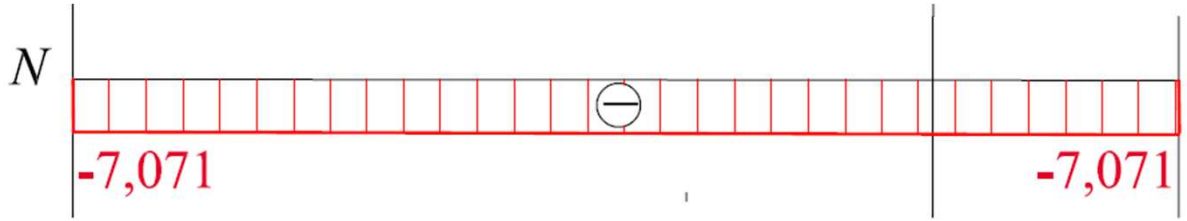
PROSTÝ NOSNÍK S PŘEVISLÝM KONCEM

2) Normálové síly



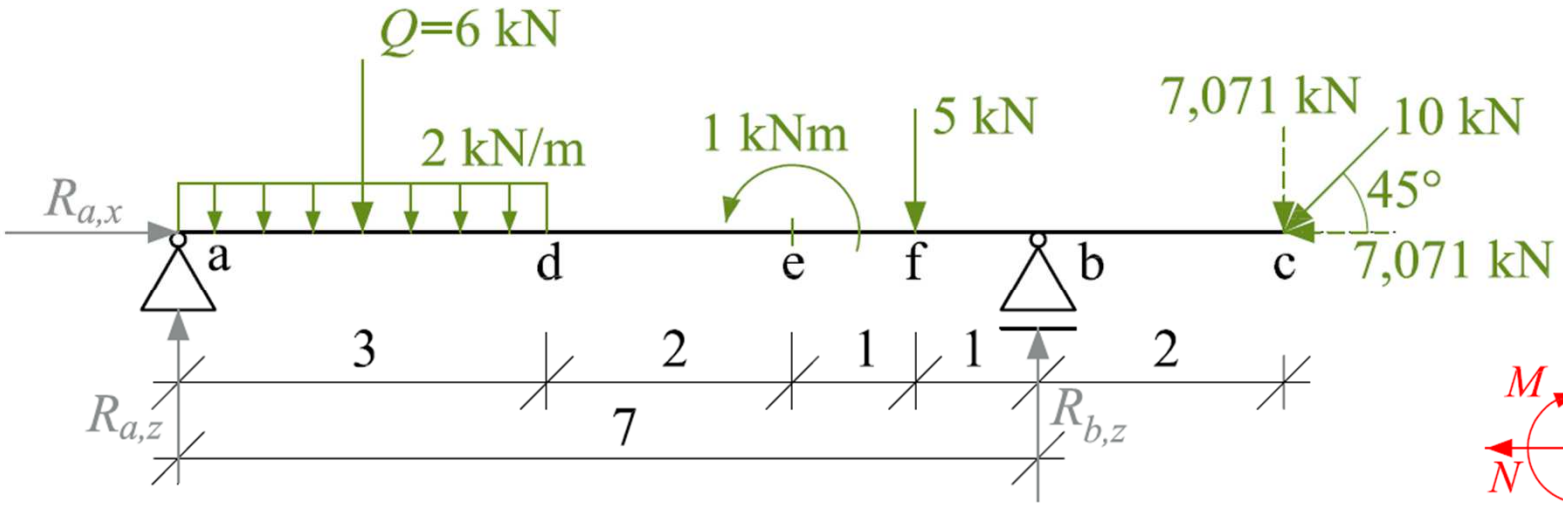


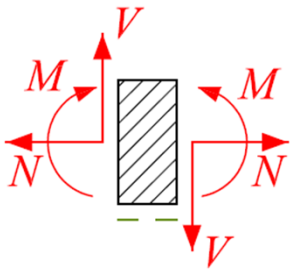
- $N_a^L = R_{a,x} = -7{,}071\ \text{kN}$
- $N_c^P = -7{,}071\ \text{kN}$



PROSTÝ NOSNÍK S PŘEVISLÝM KONCEM

3) Posouvající síly

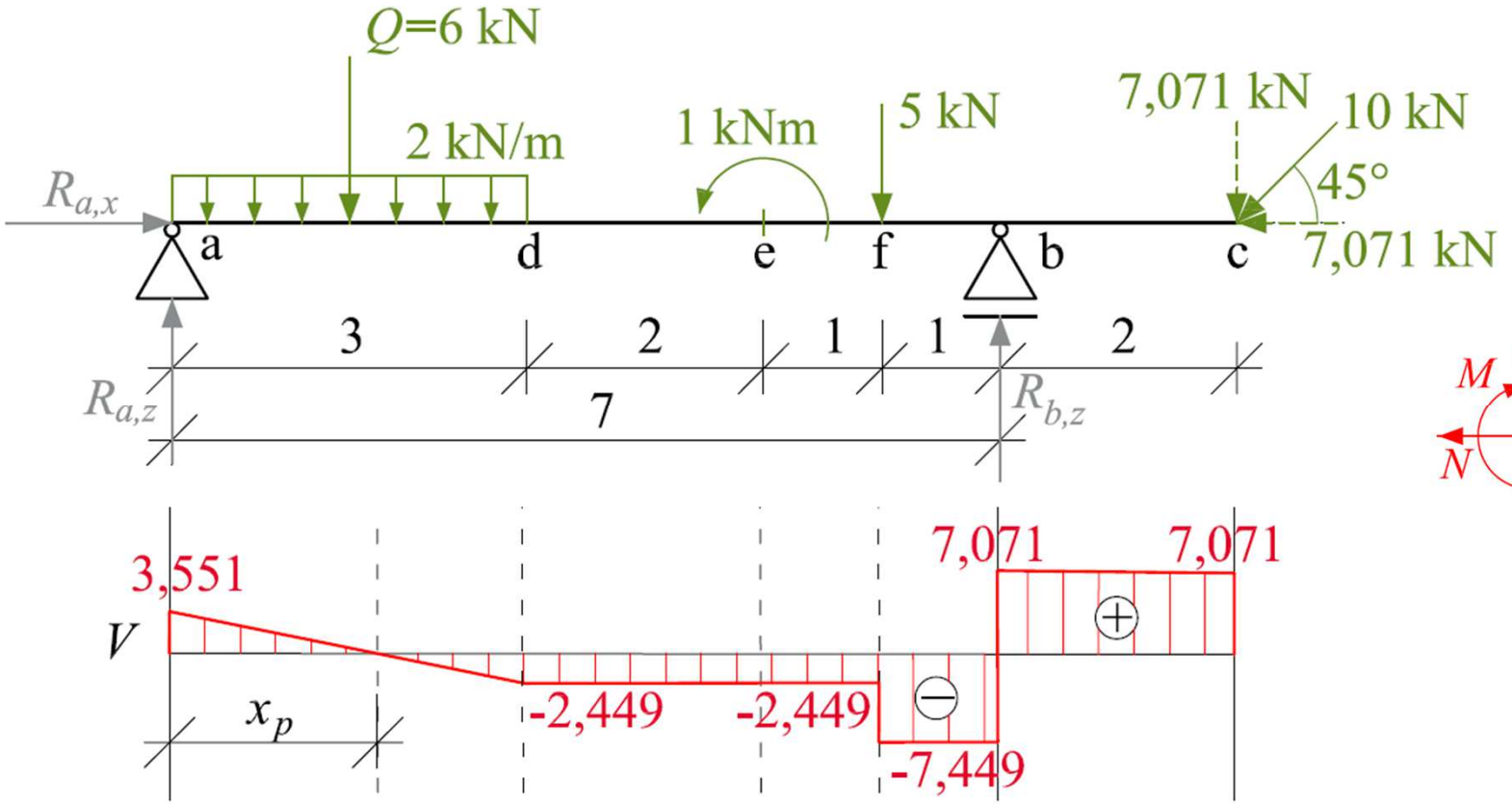


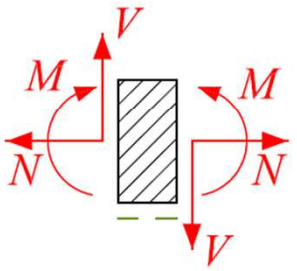


- $V_a^L = R_{a,z} = 3{,}551\ \text{kN}$
- $V_d^L = R_{a,z} - Q = 3{,}551 - 6 = -2{,}449\ \text{kN}$
- $V_{fe}^L = V_d$ (před silou 5 kN); $V_{fb}^L = V_d - 5 = -2{,}449 - 5 = -7{,}449\ \text{kN}$
- $V_c^P = 7{,}071\ \text{kN}$; $V_b^P = V_c^P - R_{b,z} = 7{,}071 - 14{,}52 = -7{,}449\ \text{kN}$

PROSTÝ NOSNÍK S PŘEVISLÝM KONCEM

3) Posouvající síly – přechodový průřez

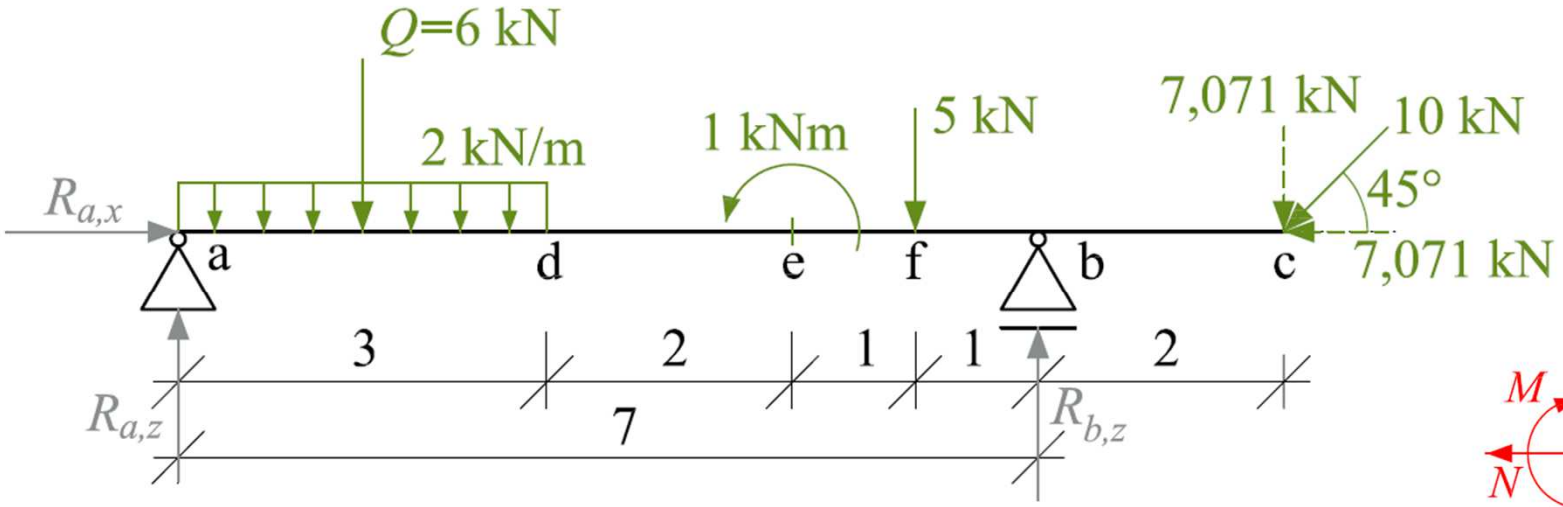


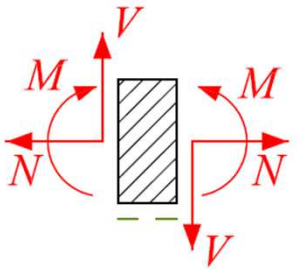


- $V_{x_p}^{L} = R_{a,z} - 2 \cdot x_p = 0; \rightarrow x_p = \frac{3{,}551}{2} = 1{,}7755\ \mathrm{m}$

PROSTÝ NOSNÍK S PŘEVISLÝM KONCEM

4) Ohybové momenty

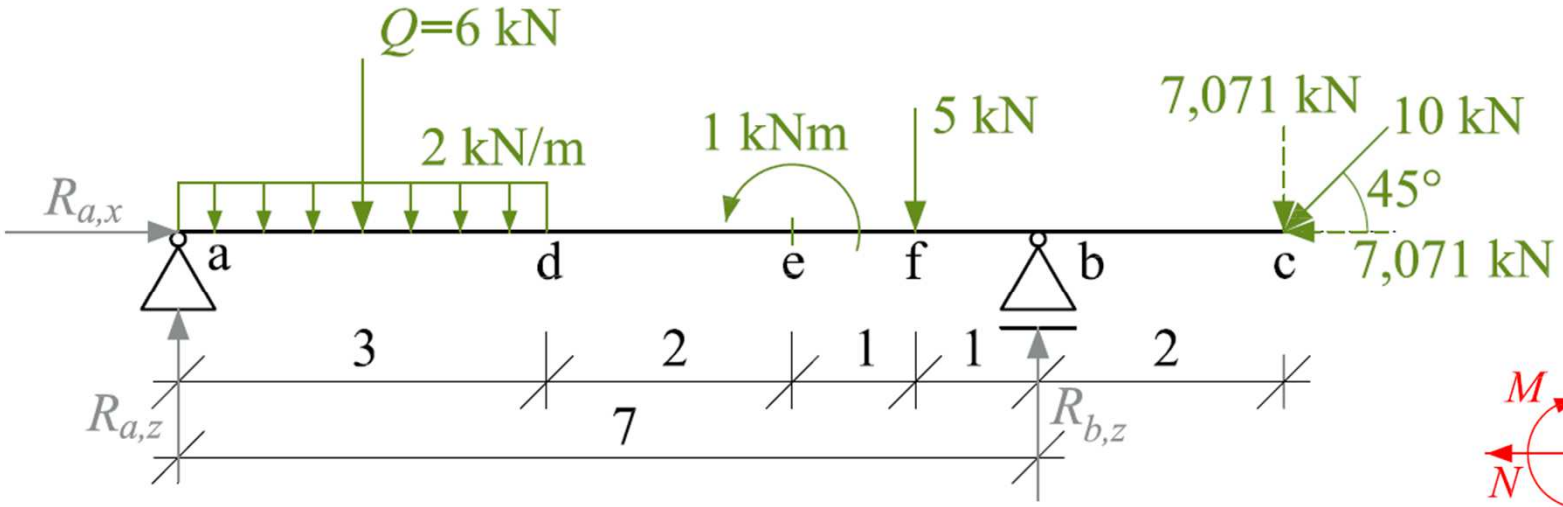


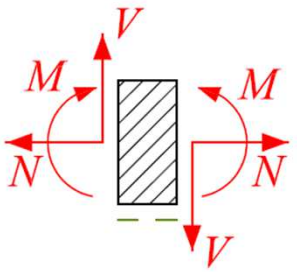


- $M_a^L = 0$
- $M_{x_p}^L = R_{a,z} \cdot x_p - 2 \cdot x_p \cdot \frac{x_p}{2} = 3{,}551 \cdot 1{,}7755 - 2 \cdot \frac{1{,}7755^2}{2} = 3{,}152\ \text{kNm}$
- $M_d^L = 3{,}551 \cdot 3 - 2 \cdot 3 \cdot \frac{3}{2} = 1{,}653\ \text{kNm}$
- $M_{ed}^L = 3{,}551 \cdot 5 - 2 \cdot 3 \cdot 3{,}5 = -3{,}245\ \text{kNm};\ M_{ef}^L = M_{ed}^L - 1 = -4{,}245\ \text{kNm}$

PROSTÝ NOSNÍK S PŘEVISLÝM KONCEM

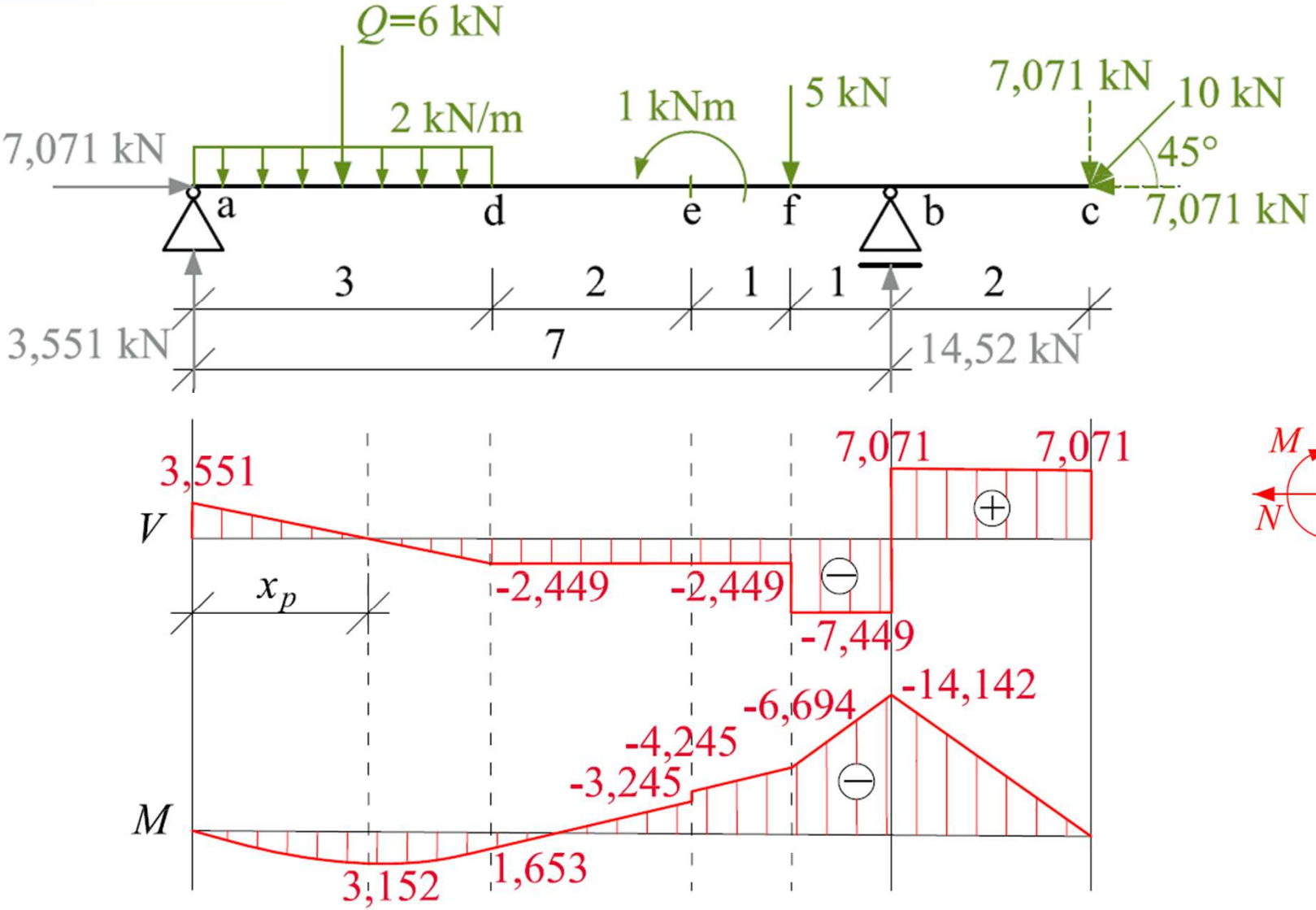
4) Ohybové momenty

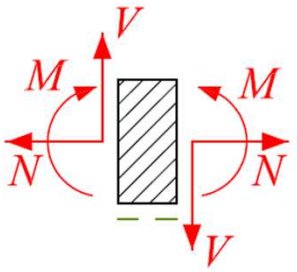




- $M_f^L = 3{,}551 \cdot 6 - 2 \cdot \ 3 \cdot 4{,}5 - 1 = -6{,}694\ \text{kNm}$
- $M_f^P = -7{,}071 \cdot 3 + 14{,}52 \cdot 1 = -6{,}693\ \text{kNm}$
- $M_b^P = -7{,}071 \cdot 2 = -14{,}142\ \text{kNm}$
- $M_c^P = 0$

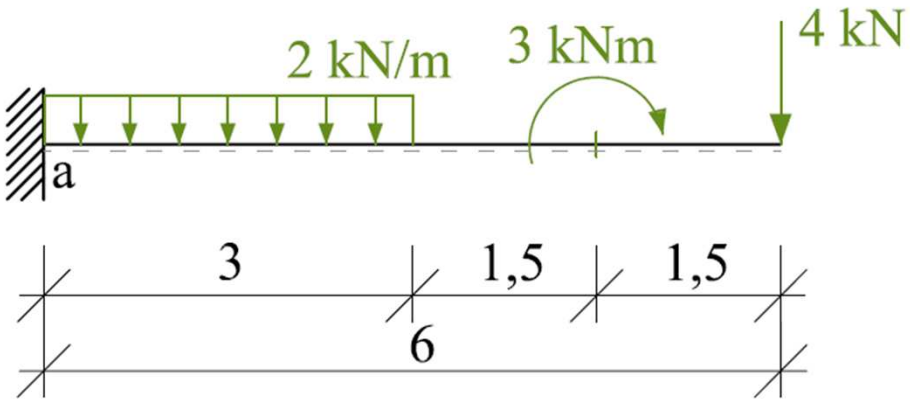
PROSTÝ NOSNÍK S PŘEVISLÝM KONCEM





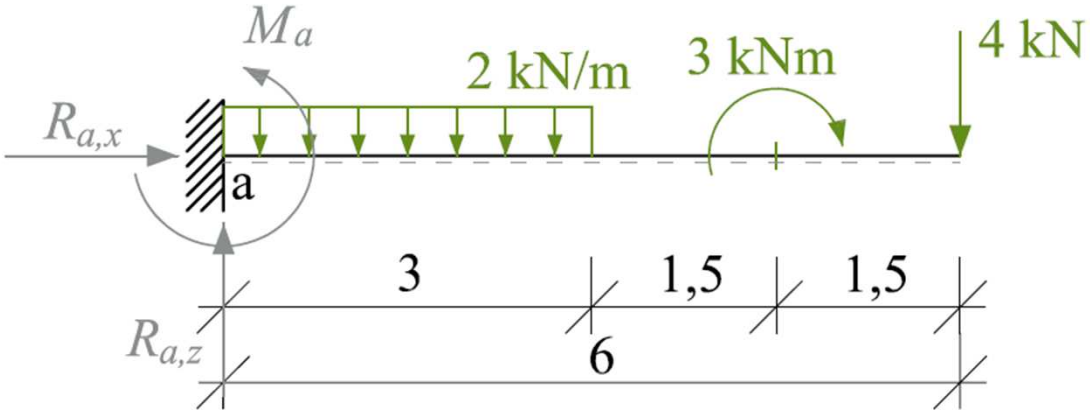
ROVINNÝ KONZOLOVÝ NOSNÍK

Vykreslete průběhy vnitřních sil



ROVINNÝ KONZOLOVÝ NOSNÍK

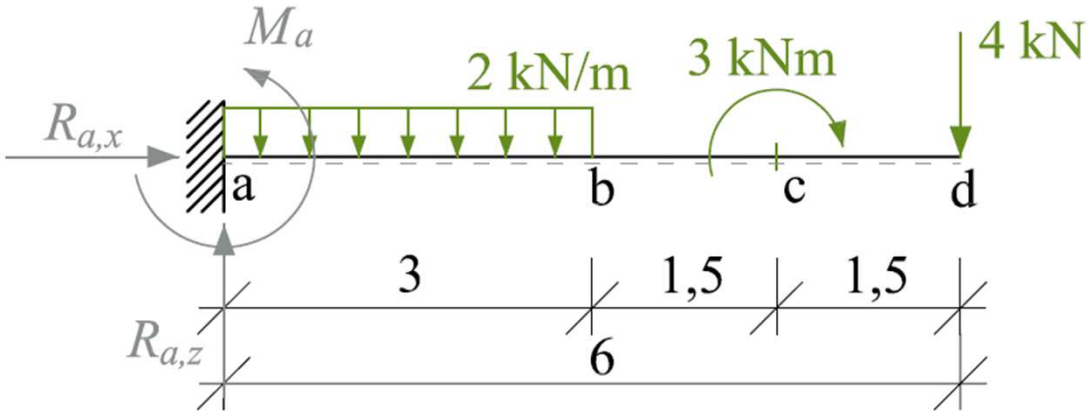
1) Výpočet reakcí

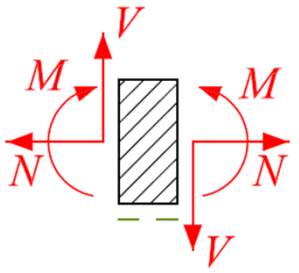


- $\sum F_{i,x} = 0 \rightarrow R_{a,x} = 0$ ⊕ →
- $\sum M_{i,a} = 0;\ M_a - 2 \cdot 3 \cdot 1{,}5 - 3 - 4 \cdot 6 = 0 \rightarrow M_a = 36\ \text{kNm}$ ⊕
- $\sum F_{i,z} = 0;\ -R_{a,z} + 2 \cdot 3 + 4 = 0 \rightarrow R_{a,z} = 10\ \text{kN}$ ⊕ ↓
- $\sum M_{i,b} = 0; 36 - 10 \cdot 6 + 2 \cdot 3 \cdot 4{,}5 - 3 = 0; 0 = 0 \rightarrow$ VYHOVÍ

ROVINNÝ KONZOLOVÝ NOSNÍK

2) Posouvající síly

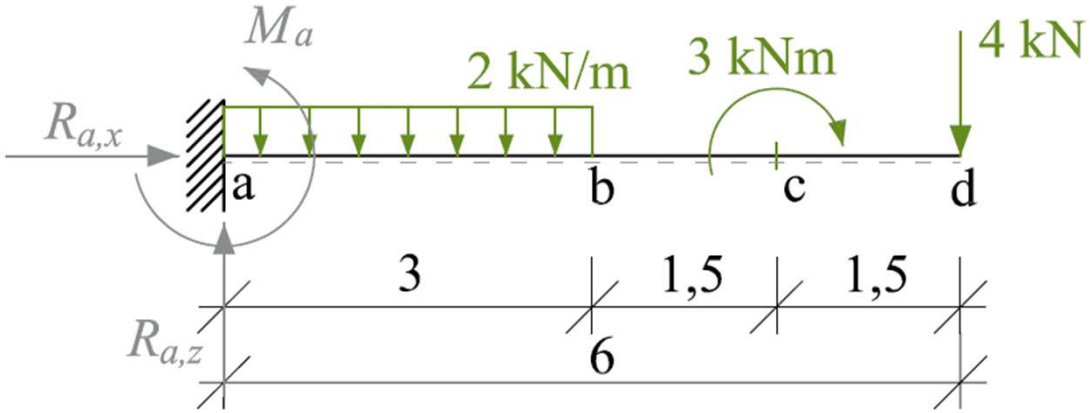


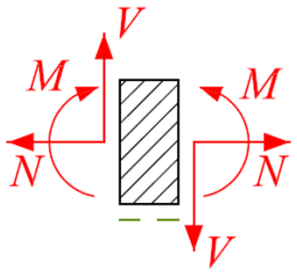


- $V_a^L = R_{a,z} = 10\ \text{kN}$
- $V_b^L = R_{a,z} - 2 \cdot 3 = 4\ \text{kN}$
- $V_d^L = V_b^L$
- $V_d^P = 4\ \text{kN}$

ROVINNÝ KONZOLOVÝ NOSNÍK

3) Ohybové momenty





- $M_a^L = -M_a = -36\ \text{kNm}$
- $M_b^L = -M_a + R_{a,z} \cdot 3 - 2 \cdot 3 \cdot 1{,}5 = -36 + 10 \cdot 3 - 2 \cdot 3 \cdot 1{,}5 = -15\ \text{kNm}$
- $M_c^L = -M_a + R_{a,z} \cdot 4{,}5 - 2 \cdot 3 \cdot 3 = -36 + 10 \cdot 4{,}5 - 2 \cdot 3 \cdot 3 = -9\ \text{kNm}$
- $M_c^P = -4 \cdot 1{,}5 = -6\ \text{kNm}$
- $M_d^P = 0$

ROVINNÝ KONZOLOVÝ NOSNÍK