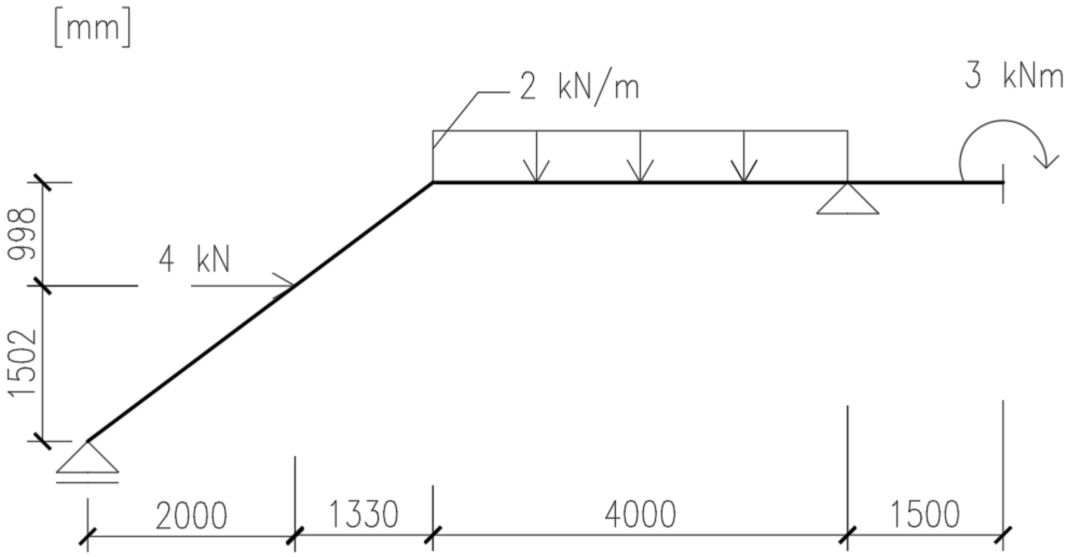
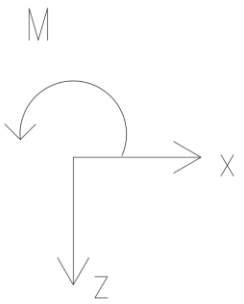
Cvičení 12

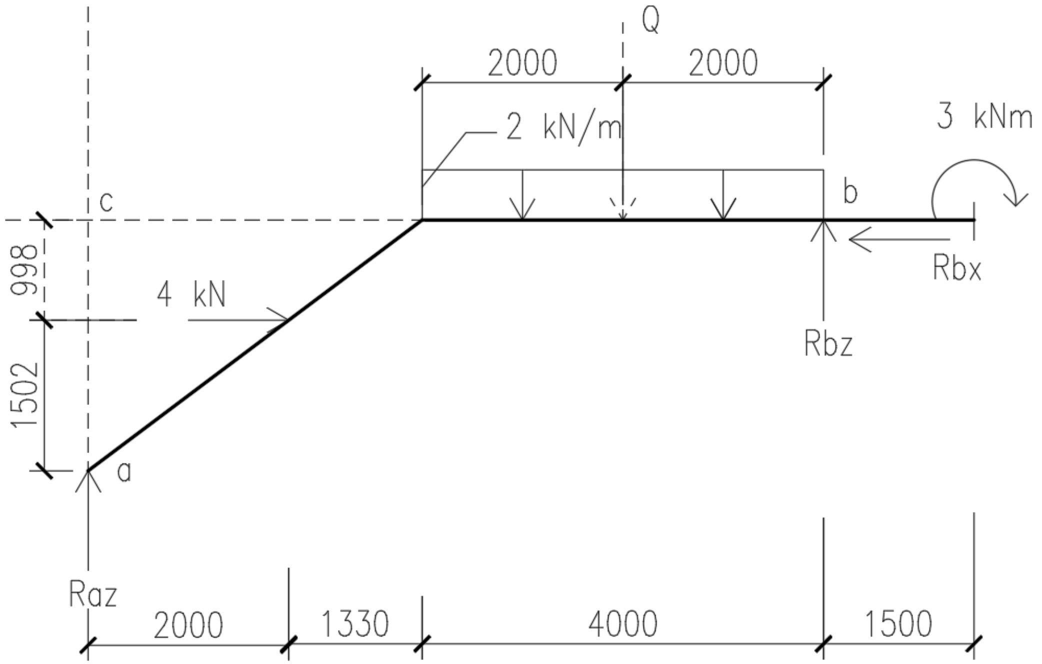
Rovinný lomený nosník se šikmými pruty, reakce a diagramy vnitřních sil a momentů.

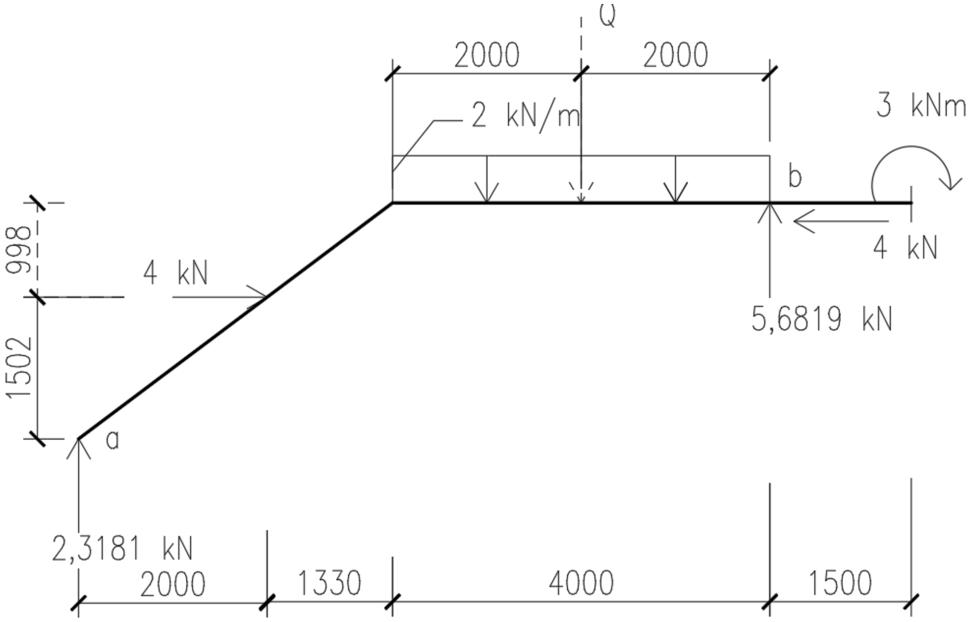
Příklad 1

- Nejprve nosník uvolníme z vazeb a ty nahradíme **předpoklady** průběhu reakcí, používáme přitom dohodnutou osovou konvenci – kladný směr je ve směru šipek









- Vypočítáme **náhradní břemeno** Q a jeho působiště vložíme do místa **těžiště** zatěžovacího obrazce

$$Q = 2 \cdot 4 = 8\, kN$$

- Libovolnou kombinací silových a momentových podmínek tyto neznámé složky reakcí vypočteme

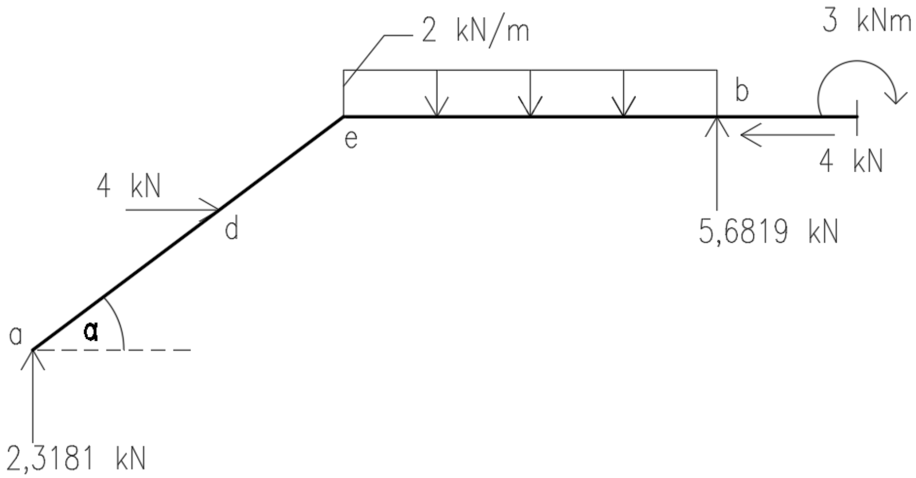
$$\sum M_{ib} = 0 \rightarrow -Raz \cdot 7{,}33 + 4 \cdot 0{,}998 + 8 \cdot 2 - 3 = 0$$

$$\rightarrow Raz = 2{,}3181\, kN \uparrow$$

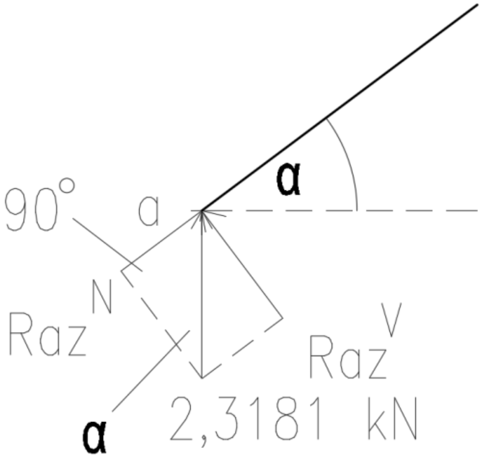
$$\sum M_{ic} = 0 \rightarrow Rbz \cdot 7{,}33 + 4 \cdot 0{,}998 - 8 \cdot 5{,}33 - 3 = 0$$

$$\rightarrow Rbz = 5{,}6819\, kN \uparrow$$

$$\sum F_{ix} = 0 \rightarrow -Rbx + 4 = 0 \rightarrow Rbx = 4\, kN \leftarrow$$



- Provedeme rozklad sil
- Rozklad sil v bodě a:



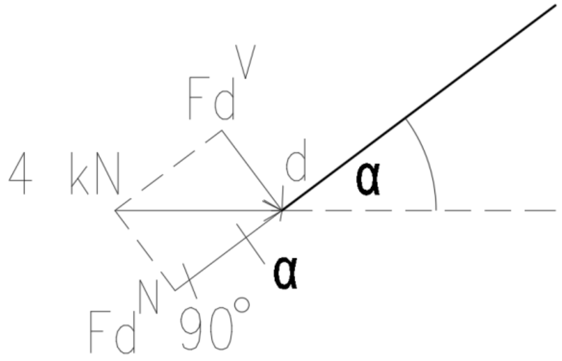
$$L = \sqrt{3{,}33^2 + 2{,}5^2} = \sqrt{17{,}3389}\, m$$

$$\cos\alpha = \frac{3{,}33}{\sqrt{17{,}3389}}$$

$$\sin\alpha = \frac{2{,}5}{\sqrt{17{,}3389}}$$

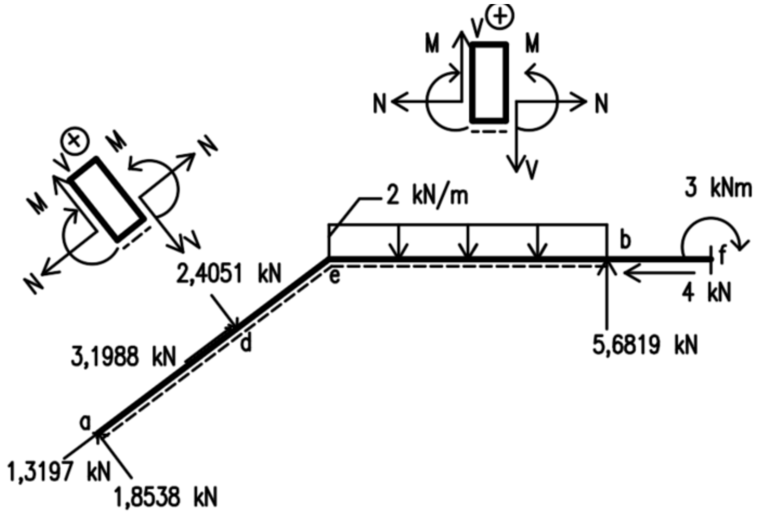
$$Raz^N = 2{,}3181 \cdot \sin\alpha = 2{,}3181 \cdot \frac{2{,}5}{\sqrt{17{,}3389}} = 1{,}3917\ kN$$

$$Raz^V = 2{,}3181 \cdot \cos\alpha = 2{,}3181 \cdot \frac{3{,}33}{\sqrt{17{,}3389}} = 1{,}8538\ kN$$



$$F^N = 4 \cdot \cos\alpha = 4 \cdot \frac{3{,}33}{\sqrt{17{,}3389}} = 3{,}1988 kN$$

$$F^V = 4 \cdot \sin\alpha = 4 \cdot \frac{2{,}5}{\sqrt{17{,}3389}} = 2{,}4015\ kN$$



- Normálové složky účinků sil:

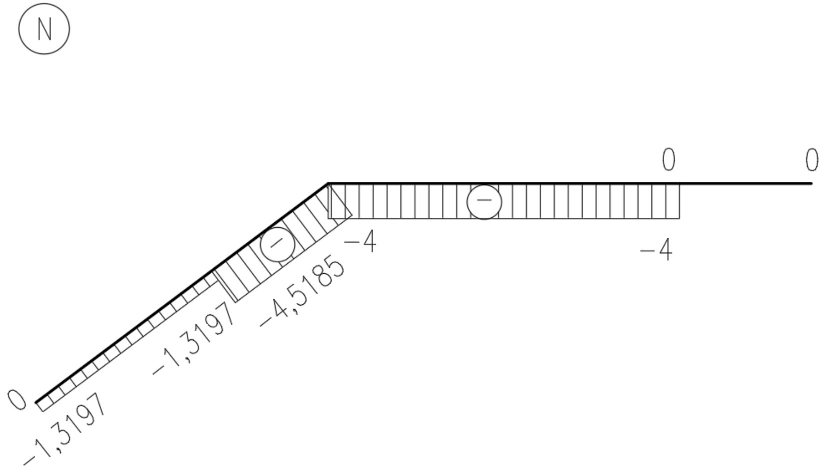
$N_a^I = 0\ kN;\ N_a^{II} = -1{,}3197\ kN$

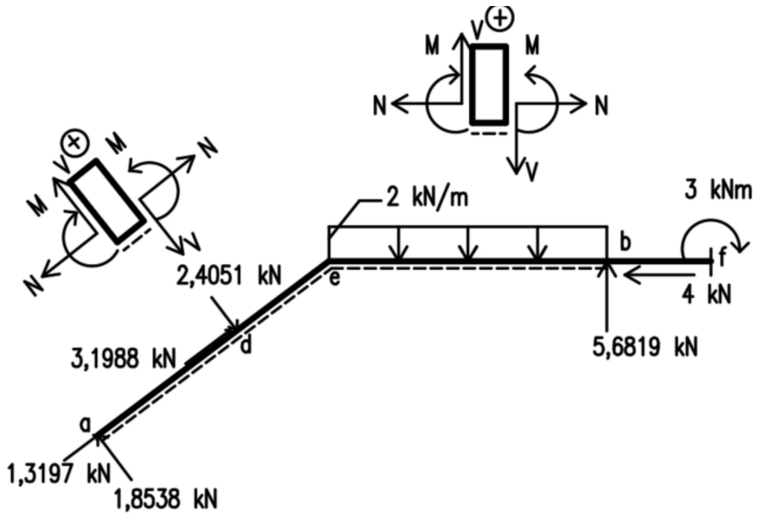
$N_d^I = -1{,}3197\ kN;\ N_d^{II} = -1{,}3197 - 3{,}1988 = -4{,}5185 kN$

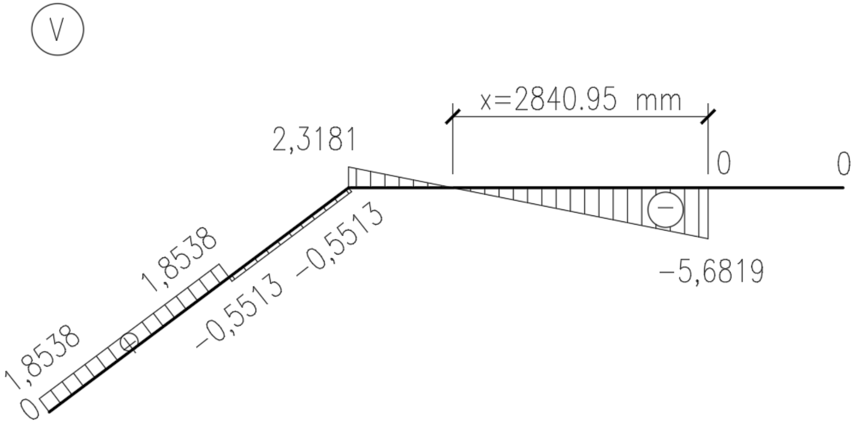
$N_e^I = -1{,}3197 - 3{,}1988 = -4{,}5185\ kN; N_e^{II} = -4\ kN$

$N_b^I = -4\ kN; N_b^{II} = -4 + 4 = 0\ kN$

$N_f^I = -4 + 4 = 0 kN; N_f^{II} = -4 + 4 = 0\ kN$







- Posouvající složky účinků sil:

$$V_a^I = 0\ kN; V_a^{II} = 1{,}8538\ kN$$

$$V_d^I = 1{,}8538\ kN; V_d^{II} = 1{,}8538 - 2{,}4051 = -0{,}5513\ kN$$

$$V_e^I = 1{,}8538 - 2{,}4051 = -0{,}5513\ kN; V_e^{II} = 2{,}3181\ kN$$

$$V_b^I = 2{,}3181 - 2 \cdot 4 = -5{,}6819\ kN$$

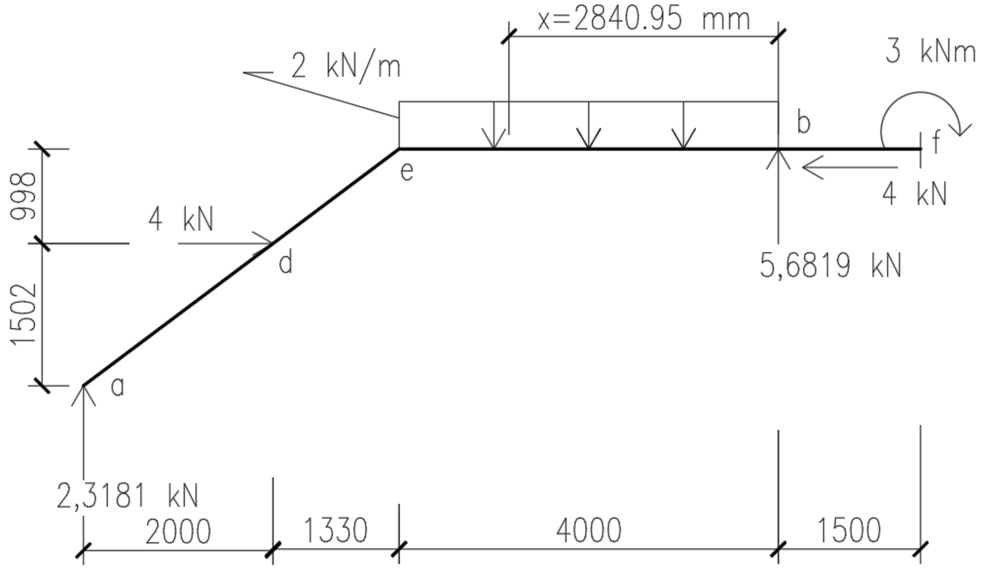
$$V_b^{II} = 2{,}3181 - 2 \cdot 4 + 5{,}6819 = 0\ kN$$

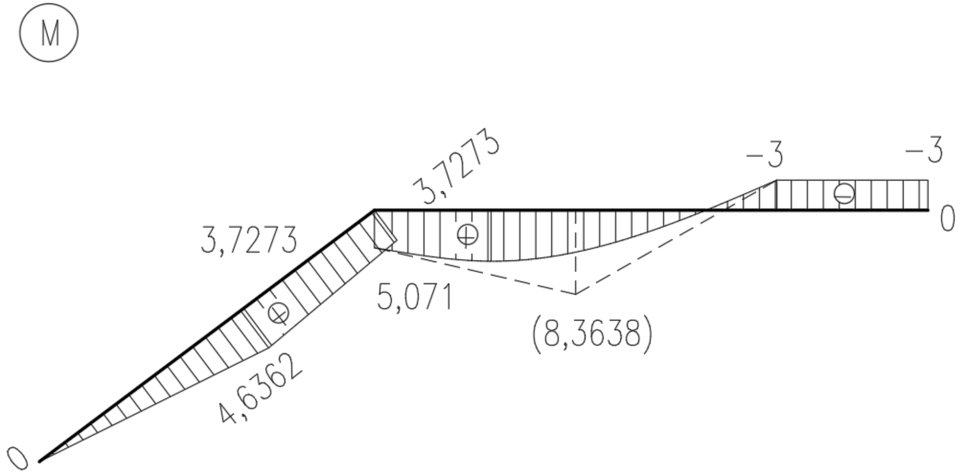
$$V_f^I = 2{,}3181 - 2 \cdot 4 + 5{,}6819 = 0\ kN$$

$$V_f^{II} = 2{,}3181 - 2 \cdot 4 + 5{,}6819 = 0\ kN$$

- Protože se na prutu nachází přechodový průřez, určíme jeho polohu

$$\frac{x}{5{,}6819} = \frac{4}{(5{,}6819 + 2{,}3181)} \rightarrow x = 2{,}84095\ m$$





- Momentové složky účinků sil:

$$M_a^I = 0\ kNm; M_a^{II} = 0\ kNm$$

$$M_d^I = 2{,}3181 \cdot 2 = 4{,}6362\ kNm; M_d^{II} = 2{,}3181 \cdot 2 = 4{,}6362\ kNm$$

$$M_e^I = 2{,}3181 \cdot 3{,}33 - 4 \cdot 0{,}998 = 3{,}7273\ kNm$$

$$M_e^{II} = 2{,}3181 \cdot 3{,}33 - 4 \cdot 0{,}998 = 3{,}7273\ kNm$$

$$M_b^I = 2{,}3181 \cdot 7{,}33 - 4 \cdot 0{,}998 - 2 \cdot \frac{4^2}{2} = -3{,}0003 \approx -3\ kNm$$

$$M_b^{II} = 2{,}3181 \cdot 7{,}33 - 4 \cdot 0{,}998 - 2 \cdot \frac{4^2}{2} = -3{,}0003 \approx -3\ kNm$$

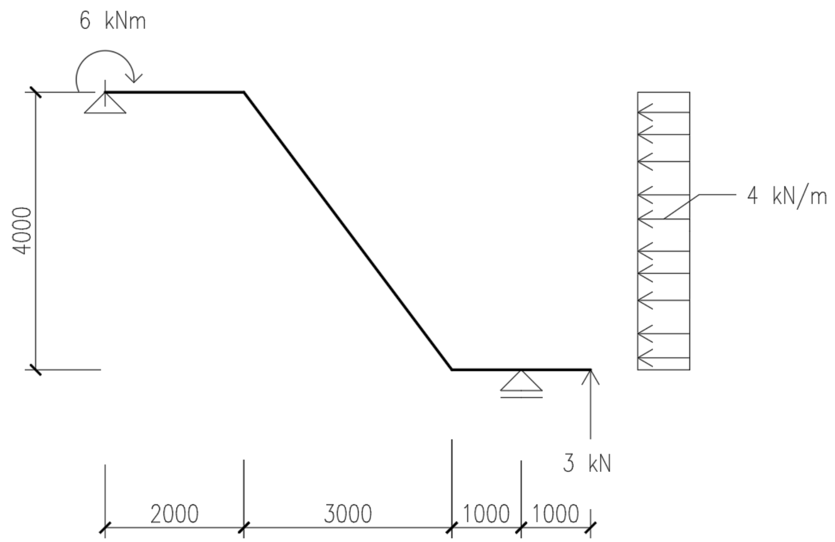
$$M_f^I = 2{,}3181 \cdot 8{,}83 - 4 \cdot 0{,}998 - 2 \cdot 4 \cdot 3{,}5 + 5{,}6819 \cdot 1{,}5 = -3{,}0003 \approx -3\ kNm$$

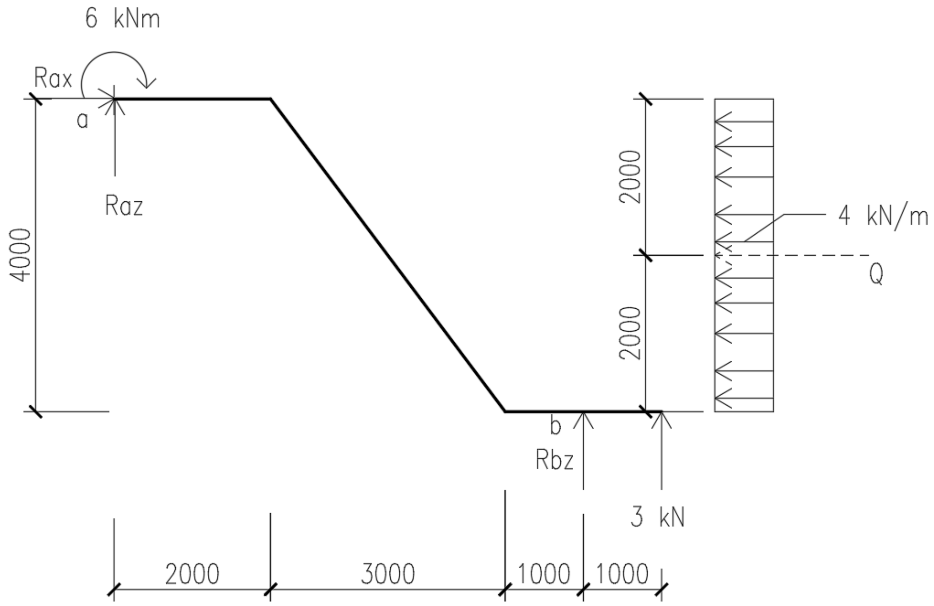
$$M_f^{II} = 2{,}3181 \cdot 8{,}83 - 4 \cdot 0{,}998 - 2 \cdot 4 \cdot 3{,}5 + 5{,}6819 \cdot 1{,}5 + 3 = -0{,}0003 \approx -0\ kNm$$

$$M_{max} = -3 + 5{,}6819 \cdot 2{,}84095 - 2 \cdot \frac{2{,}84095^2}{2} = 5{,}071\ kNm$$

$$M_0 = -3 + 5{,}6819 \cdot 2 = 8{,}3638\ kNm$$

Příklad 2



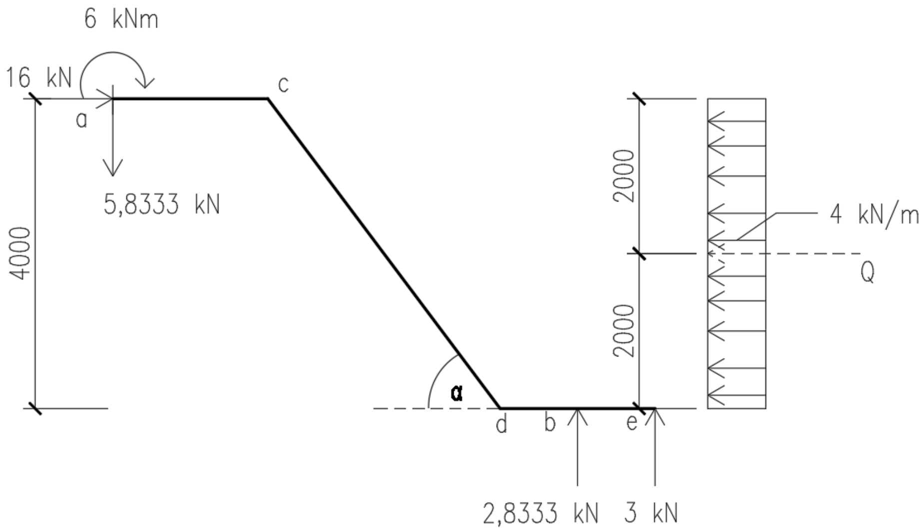


$$Q = 4 \cdot 4 = 16\ kN$$

$$\sum M_{ia} = 0 \rightarrow Rbz \cdot 6 + 3 \cdot 7 - 16 \cdot 2 - 6 = 0 \rightarrow Rbz = 2{,}8333\ kN \uparrow$$

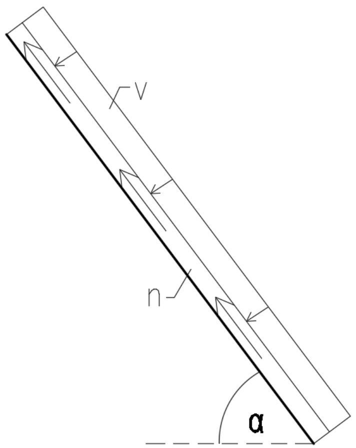
$$\sum F_{ix} = 0 \rightarrow Rax - 16 = 0 \rightarrow Rax = 16\ kN \rightarrow$$

$$\sum M_{ib} = 0 \rightarrow -Raz \cdot 6 - 6 - 16 \cdot 4 + 16 \cdot 2 + 3 \cdot 1 = 0 \rightarrow Raz$$
$$= 5{,}8333\ kN \downarrow$$



- Rozklad spojitého zatížení

$$Q = Q' \rightarrow 4 \cdot 4 = q' \cdot 5 \rightarrow q' = 3{,}2\ kN/m$$



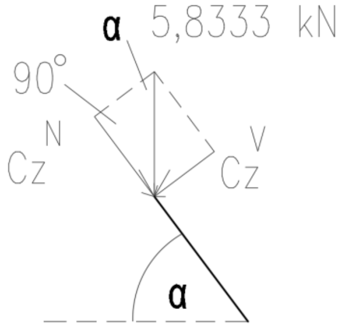
$$v = q' \cdot \sin\alpha = 2{,}56\ kN/m$$

$$n = q' \cdot \cos\alpha = 1{,}92\ kN/m$$

- Provedeme rozklad sil

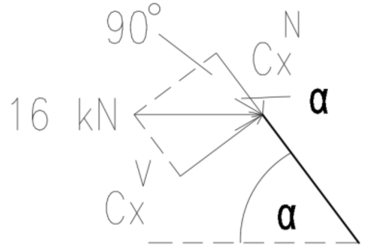
$$L = \sqrt{4^2 + 3^2} = 5\ m \qquad \cos\alpha = \frac{3}{5} \qquad \sin\alpha = \frac{4}{5}$$

- Bod c:



$$C_z^N = 5{,}8333 \cdot \sin\alpha = 5{,}8333 \cdot \frac{4}{5} = 4{,}6664\ kN$$

$$C_z^V = 5{,}8333 \cdot \cos\alpha = 5{,}8333 \cdot \frac{3}{5} = 3{,}4998\ kN$$

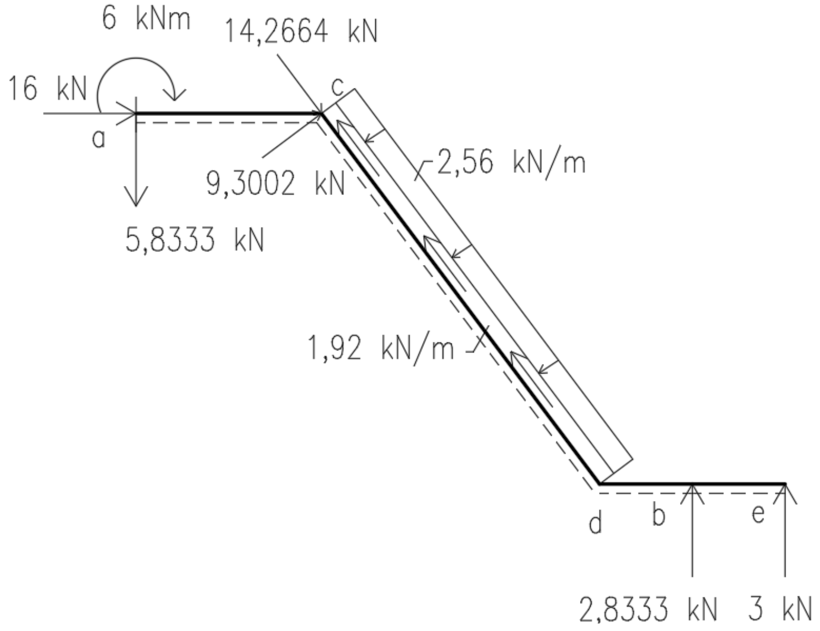


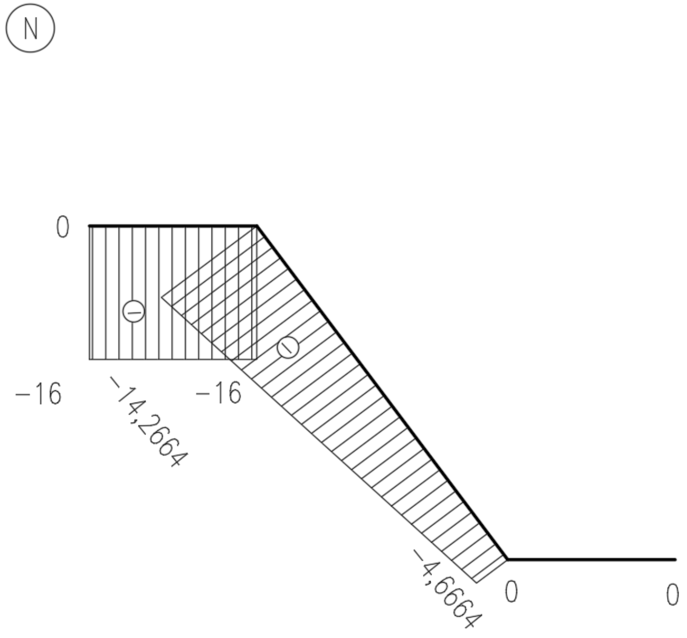
$$C_x^N = 16 \cdot \cos\alpha = 16 \cdot \frac{3}{5} = 9{,}6\ kN$$

$$C_x^V = 16 \cdot \sin\alpha = 16 \cdot \frac{4}{5} = 12{,}8kN$$

$$Nc = 4{,}6664 + 9{,}6 = 14{,}2664\ kN$$

$$\mathrm{V}c = -3{,}4998 + 12{,}8 = 9{,}3002\ kN$$





- Normálové složky účinků sil:

$$N_a^I = 0\ kN;\ N_a^{II} = -16\ kN$$

$$N_c^I = -16\ kN;\ N_c^{II} = -14{,}2664\ kN$$

$$N_d^I = -14{,}2664 + 1{,}92 \cdot 5 = -4{,}6664\ kN$$

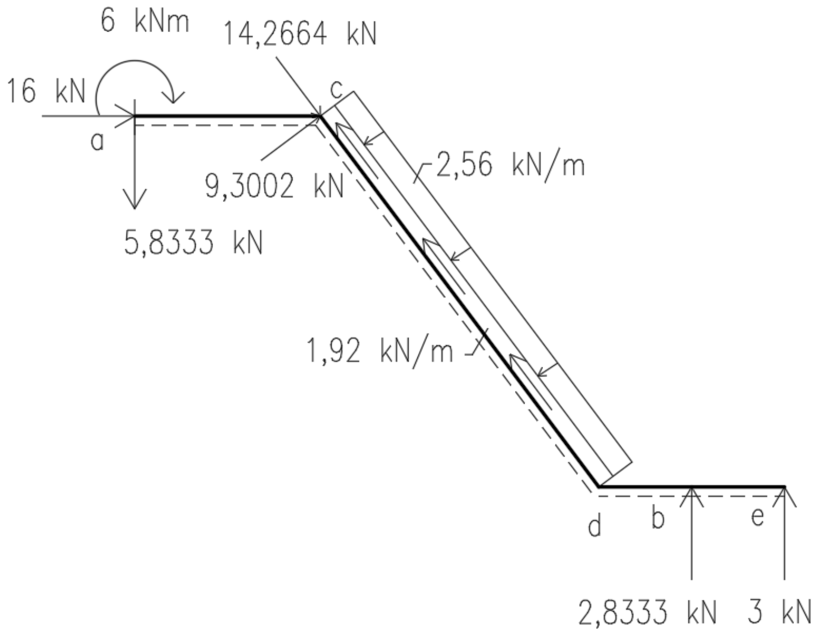
$$N_d^{II} = -16 + 4 \cdot 4 = 0\ kN$$

$$N_b^I = -16 + 4 \cdot 4 = 0\ kN$$

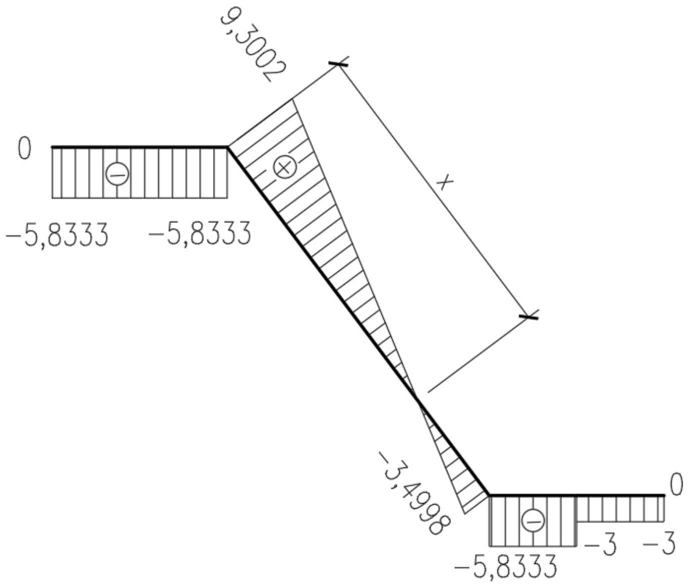
$$N_b^{II} = -16 + 4 \cdot 4 = 0\ kN$$

$$N_e^I = -16 + 4 \cdot 4 = 0\ kN$$

$$N_e^{II} = -16 + 4 \cdot 4 = 0\ kN$$



V



- Posouvající složky účinků sil:

$$V_a^I = 0\ kN;\ V_a^{II} = -5{,}8333\ kN$$

$$V_c^I = -5{,}8333\ kN;\ V_c^{II} = 9{,}3002\ kN$$

$$V_d^I = 9{,}3002 - 2{,}56 \cdot 5 = -3{,}4998\ kN$$

$$V_d^{II} = -5{,}8333\ kN$$

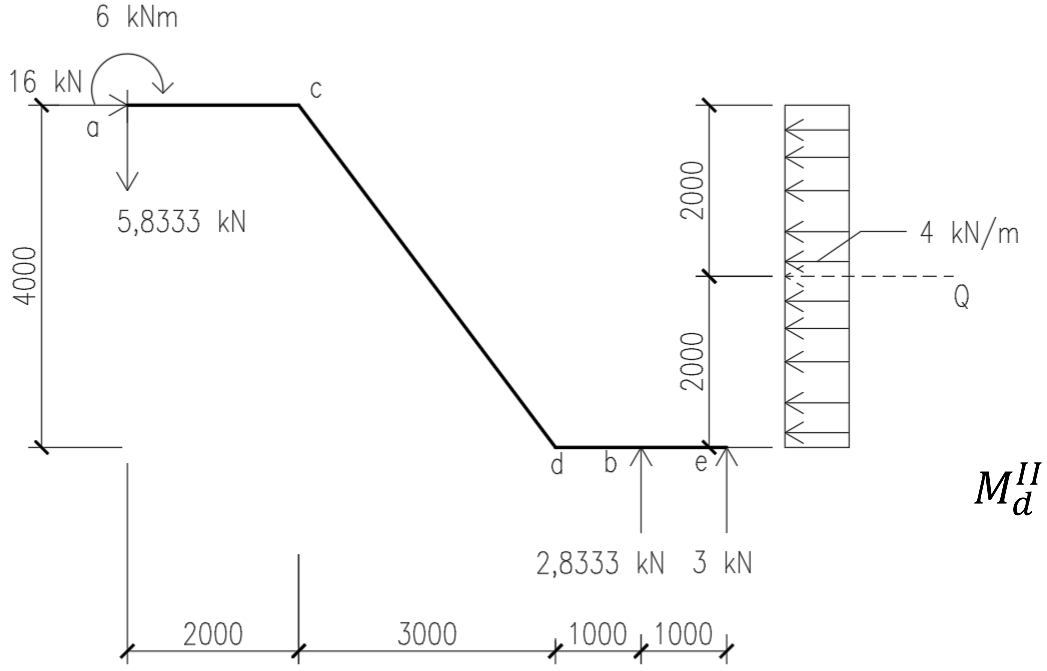
$$V_b^I = -5{,}8333\ kN$$

$$V_b^{II} = -5{,}8333 + 2{,}8333 = -3\ kN$$

$$V_e^I = -5{,}8333 + 2{,}8333 = -3\ kN$$

$$V_e^I = -5{,}8333 + 2{,}8333 + 3 = 0\ kN$$

$$\frac{x}{9{,}3002} = \frac{5}{(9{,}3002 + 3{,}4998)} \rightarrow x = 3{,}6329\ m$$



- Momentové složky účinků sil:

$$M_a^I = 0\,kNm;\; M_a^{II} = 6\,kNm$$

$$M_c^I = 6\, - 5{,}8333 \cdot 2 = -5{,}6666\,kNm$$

$$M_c^{II} = 6\, - 5{,}8333 \cdot 2 = -5{,}6666\,kNm$$

$$M_d^I = 6\, - 5{,}8333 \cdot 5 + 16 \cdot 4 - 4 \cdot \frac{4^2}{2} = 8{,}8335\,kNm$$

$$M_d^{II} = 6\, - 5{,}8333 \cdot 5 + 16 \cdot 4 - 4 \cdot \frac{4^2}{2} = 8{,}8335\,kNm$$

$$M_b^I = 6\, - 5{,}8333 \cdot 6 + 16 \cdot 4 - 4 \cdot \frac{4^2}{2} = 3{,}0002 \approx 3\,kNm$$

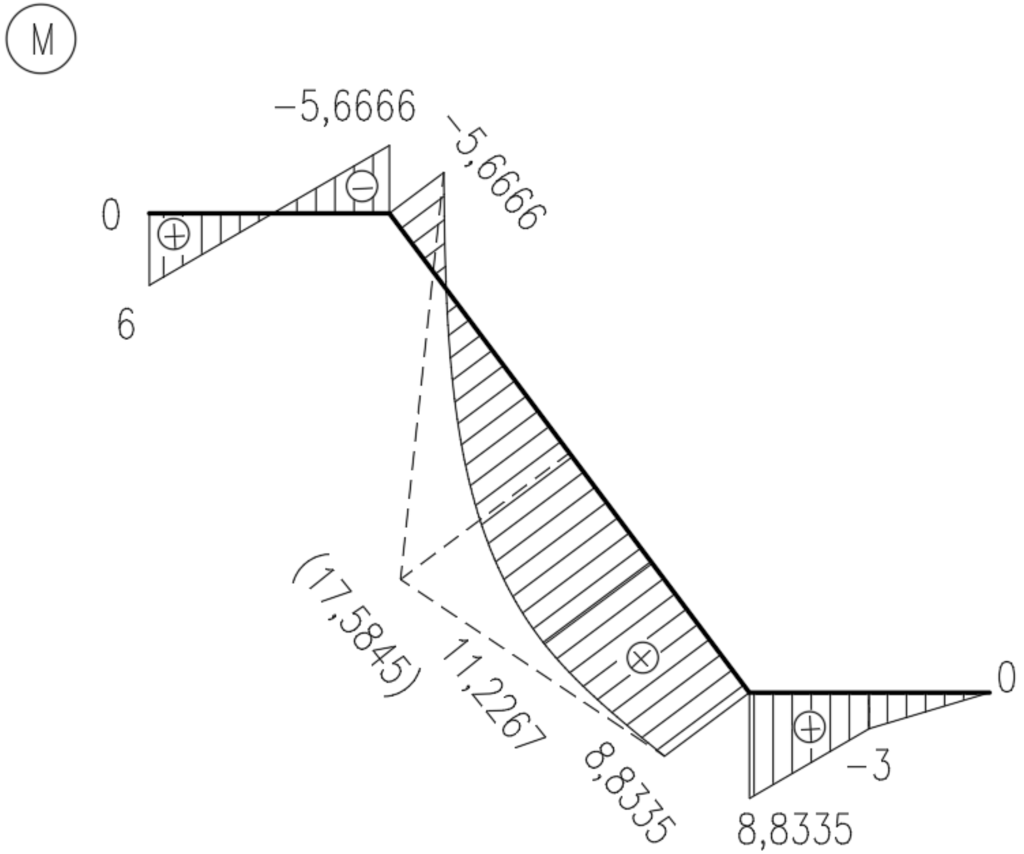
$$M_b^{II} = 6\, - 5{,}8333 \cdot 6 + 16 \cdot 4 - 4 \cdot \frac{4^2}{2} = 3{,}0002 \approx 3\,kNm$$

$$M_e^I = 6\, - 5{,}8333 \cdot 7 + 16 \cdot 4 - 4 \cdot \frac{4^2}{2} + 2{,}8333 \cdot 1 = 0{,}0002 \approx 0\,kNm$$

$$M_e^{II} = 6\, - 5{,}8333 \cdot 7 + 16 \cdot 4 - 4 \cdot \frac{4^2}{2} + 2{,}8333 \cdot 1 = 0{,}0002 \approx 0\,kNm$$

- Díky znalosti **Schwedlerovy věty** – **diferenciálních podmínek rovnováhy**, snadno získáme velikost M_{max}:

$$M_{max} = M_c^{II} + \int_c^x V dx = -\,5{,}6666 + \frac{9{,}3002 \cdot 3{,}6329}{2} = 11{,}2267\,kNm$$



- Z podobnosti trojúhelníků nejdeme polohu těžiště zatěžovacího obrazce

$$\frac{x'}{2,5} = \frac{3}{5} \rightarrow x' = 1,5\ m \qquad \frac{y'}{2,5} = \frac{4}{5} \rightarrow y' = 2\ m$$

$$M_0 = 6\ - 5,8333 \cdot (2 + 1,5) + 16 \cdot 2 = 17,5845\ kNm$$