Size effects in concrete specimens under uniaxial tension: Computational modeling

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Abstract
This paper presents a study of numerical modeling of sources of size effects in concrete structures published in International Journal of Fracture [3]. The major motivation is to identify and study interplay of several scaling length stemming from the material, boundary conditions and geometry. We compare our results with the well published results of direct tensile tests of dog-bone specimen with rotating boundary conditions. The specimens are modeled using microplane material and fracture-plastic material law to show that a portion of the dependence of nominal strength on structural size can be explained deterministically. We model individual sources of size effect. Namely, we model local material strength using an autocorrelated random field to identify a statistical part of size effect. Another size effect sources could be explained either by the presence of a weak surface layer of constant thickness (caused e.g. by drying, surface damage, or other irregularities) and three dimensional out-of-plane specimen flexure. The latter effect is examined on 3D models in comparison with 2D models with the same material law. All three sources of size effect (deterministic-energetic, statistical effects and the weak layer effect) are believed to be the sources most contributing to the observed strength size effect; the model combining all of them is capable of reproducing the measured data. We use methods of advanced computational nonlinear fracture mechanics with simulation techniques for random fields representing material properties. We show how different sources of size effects detrimental to strength can interact and result in relatively complicated quasibrittle failure processes.

1. Introduction

The paper studies interacting size effects on the nominal strength of concrete structures. The main target is to identify possible sources of size effect, study them and model them together in one complex model using a combination of finite element software enabling nonlinear analyses and probabilistic methods.

For this purpose, we used the well published experimental results of direct tensile tests on dog-bone shaped specimens with rotating boundary conditions of varying size (size range 1:32) performed by van Vliet and van Mier. We were interested in the series of "dry" concrete specimens A to F (dimension D varying from 50 to 1,600 mm). All specimens were kept the constant width $b = 100$ mm and were geometrically similar. Specimens were loaded in uniaxial tension with geometrically scaled eccentricity $e = D/50$. The paper attempts to explain the interacting size effects (deterministic and stochastic size effect, "weak boundary" effect and out of plane rotation of stiffness plate).

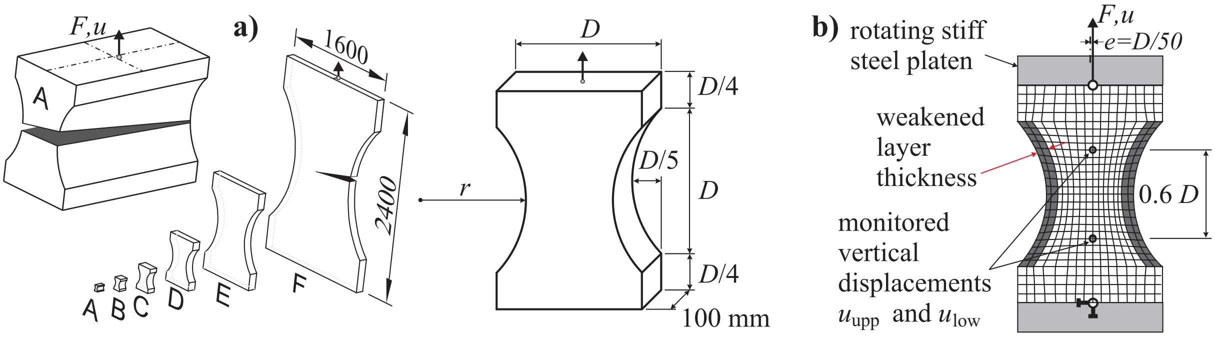

Figure 1: (a) Dog-bone series (specimens A to F) tested by van Vliet and van Mier; (b) 2D models in ATENA software with a surface layer. Strain ε is calculated using the separation $\Delta u = u_{\mathrm{upp}} - u_{\mathrm{low}}$ of two points over the control length $l_c = 0.6D$.

2. Modeling in 2D

Most of our studies were performed with 2D models prepared in ATENA program [1]. We started with microplane material model and later we compared results with fracture-plastic material model (3D Nonlinear Cementitious 2 – NLCEM).

Specimens were loaded by prescribed deformation and the force F was monitored. We ignored the transition from plane strain to plane stress conditions with growing specimen size. We modeled all specimen sizes using a plane stress model.

2.1. Deterministic-energetic

Firstly we studied "deterministic-energetic" size effect caused by an approximately constant fracture process zone (FPZ) size with stress redistribution in specimens of all sizes. We modeled this effect in non-linear finite element software ATENA containing Bažant's microplane material model M4 combined with the crack band model to eliminate dependency of the results on the mesh size.

Microplane parameters were generated by ATENA from experimentally obtained cube compressive strength 50 MPa: K1 $= 1.5644 \cdot 10^{-4}$, K2 $= 500$, K3 $= 15$, K4 $= 150$, crack band $c_{\mathrm{b}} = 30$ mm, number of microplanes 21. We changed the crack band to $c_{\mathrm{b}} = 8$ mm, a value that enables us to explain most of the experimentally obtained size effect, see the thick line with solid circles in Fig. 4. The most relevant parameters for NLCEM material are: the cube compressive strength of 50 MPa, uniaxial compressive strength $f_{\mathrm{c}} = 42.5$ MPa, modulus of elasticity $E = 36.95$ GPa (the initial stiffness of microplane models and NLCEM models are equal), uniaxial tensile strength $f_{\mathrm{t}} = 3.2$ MPa, fracture energy $G_{\mathrm{F}} = 200$ N/m (exponential crack opening law). Using this set of material parameters we have performed deterministic computations with a wide range of structure sizes.

2.2. Stochastic

We believe that the main size effect on strength is caused by the spatial variability of local material strength. Therefore, in previous study [4], we considered the strength related parameter K1 in the microplane material model in ATENA to be random, and performed Monte Carlo type simulations for each size of specimen. The same strategy was performed also with the fracture-plastic material model NLCEM, we randomized the tensile strength f_{t}. We sampled 64 random field realizations of the parameter K1 (f_{t}) for each specimen size and computed the responses. For sampling of the local material strength we use autocorrelated random field, because we believe that in reality the strength of any two close locations must be strongly related (correlated). The distribution of local tensile strength at each material point is assumed to be identical and Weibull distributed. In Fig. 2 we plot selected realizations of the random strength field (top surface) in NLCEM for all sizes A - F. The middle surface shows the maximum principal stress field at the peak load if no redistribution takes place and when stress could exceed the local strength. The bottom surface is the actual (redistributed) stress field. A discretized random field can be viewed as set of (auto)correlated variables. The most important parametr is the autocorrelation length with its role as a measure of the rate of fluctuation of local material parameters, significantly influence the damage process and the global response of the structure.

To obtain results consistent with the previous deterministic analysis, we used the value of parameter K1 $= 1.5644 \cdot 10^{-4}$ ($f_\mathrm{t} = 3.2$ MPa) as the mean values. The second parameter of Weibull distribution with regard to the cov $= 0.16$ of the nominal strength of the smallest specimen A. This choice is supported by the fact that size A has the largest sample size (the estimation of variance has a higher statistical significance). For simplicity we use cov $= 0.15$ (15% variability of local strength), then shape parameter $m = 7.91$.

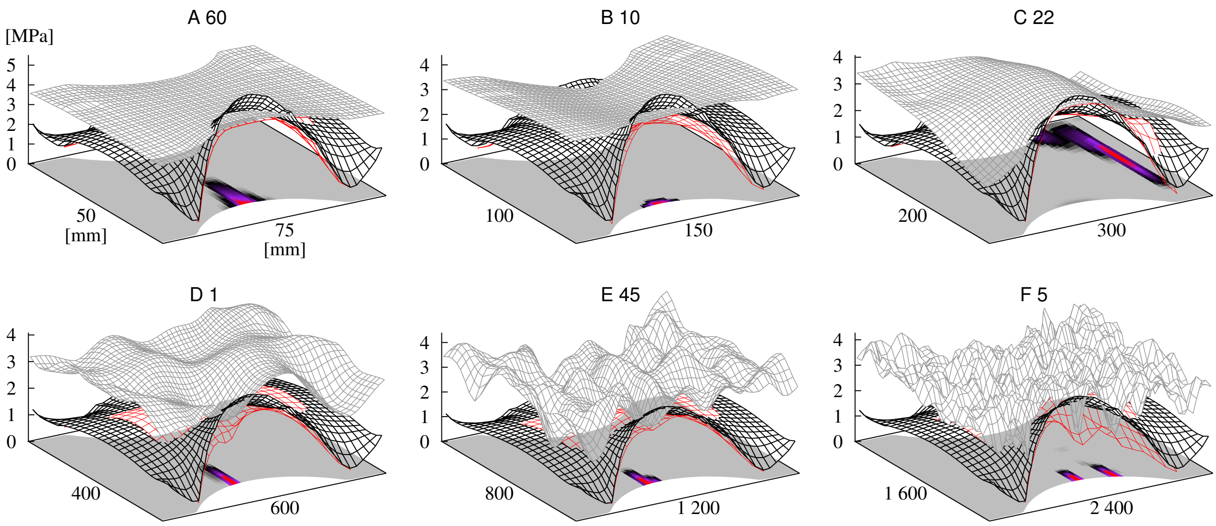

Figure 2: Stress/strength fields corresponding to the peak load for selected realizations and specimen sizes. Fields from top: random strength field (threshold), principal stress of a brittle material scaled to correspond to the peak load (nominal strength), actual principal stress at peak load, cracking strain at the bottom plane.

3. Modeling in 3D

The above described models in 2D were created assuming plane stress conditions. This simplification could be a source of an error, because the width of the small specimens is not negligible compared to other dimensions. 3D model define better the hinges (pendulum system) that could freely rotate in all directions (enabling also the out of plane rotation).

We modeled the dog-bone specimens of all sizes in 3D version of ATENA software. We used fracture-plastic material model with the same material law as in 2D models. The study was performed (i) with a uniform stiffness distribution and (ii) with a three-layer material (three different Young's moduli E).

Three-layers model non-homogeneity created by the manufacturing process (casting in three layers), see Fig. 3 right. The weighted average of the three moduli was equal to the modulus used in the homogeneous case. The three values 35.13, 30.59 and 24.93 GPa were set such that their ratios are equal to the ratios used by van Vliet and van Mier [2]. Their reasoning was as

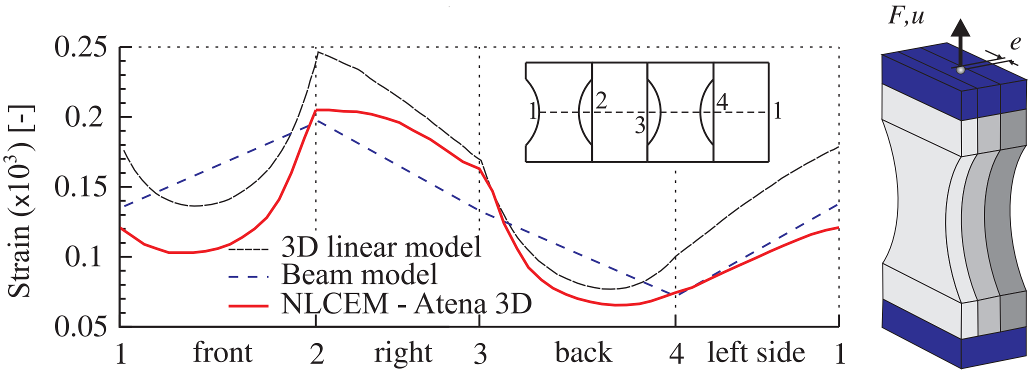

Figure 3: Left: Strain distribution along the edges of the smallest cross section of an eccentrically loaded inhomogeneous specimen. Comparison of our ATENA results with analyses by van Mier and van Vliet. Right: Three-layered inhomogeneous model in ATENA 3D software.

follows: the front face is less stiff and this causes an internal eccentricity. These layers could cause out of plane eccentricity due to their different stiffness and start to initiate cracks from the front face. Our computations with non-linear material law did not support this idea. In small size specimens is the response relative ductile. For large sizes the layered material makes no difference due to negligible specimen width to other dimensions.

Specimens of size C; its response is on the half-way of transition between the brittle and plastic limits. For specimen size C we present the computed strain profile in the pre-peak phase along the edges of the smallest cross-section (neck). Profile was compared with computations with previously obtained results of van Mier and van Vliet [2], see Fig. 3 left.

4. Analysis of the results

Fig. 4 presents the resulting nominal strengths for all sizes obtained by non-linear FEM simulations and compared to experiments (large circles with error bars). We can see that nominal strength obtained deterministically (small circles) is in transition between two asymptotes: plastic $\sigma_{\mathrm{N},0}$ (small size) and elastic $\sigma_{\mathrm{N},\infty}$ (large size). This transitional region is controlled by the characteristic length (crack band width c_b in microplane and the fracture energy G_F in NLCEM). Starting from C the dependence of mean nominal strength on size is predominantly statistical and we are not able to model it by deterministic model.

In the case of a random field description of local strength, the mean nominal strength of large sized specimens reach Weibull asymptote (see solid line obtained by Weibull integral for sizes D, E, F and G). Weibull asymptotic slope is controlled by the autocorrelation length. In our case asymptotic slope (-$n/m = -2/7.91$) is not in a good agreement with scatter of measured nominal strengths for sizes greater than A. A better choice in this study would be $m \approx 14$ suggested mainly by the slope of the size effect curve for the two largest specimen sizes E, F. When the correlation length is much larger than the specimen, the realizations of random strength are nearly constant functions and the damage spread is governed solely by the deterministic effects (see Fig. 2 A60). When the correlation length is very small compared to the nonlocal length, the damage process will depend on interactions of zones in which the damaged material softens locally. The zones with a high local strength adjacent to the weaker zones act as 'barriers' for further spreading-out of damage, see Fig. 2 F5. The size of the damaged zone depends probably strongly on the deterministic length and only weakly on the autocorrelation length of local strength field in our continuum model.

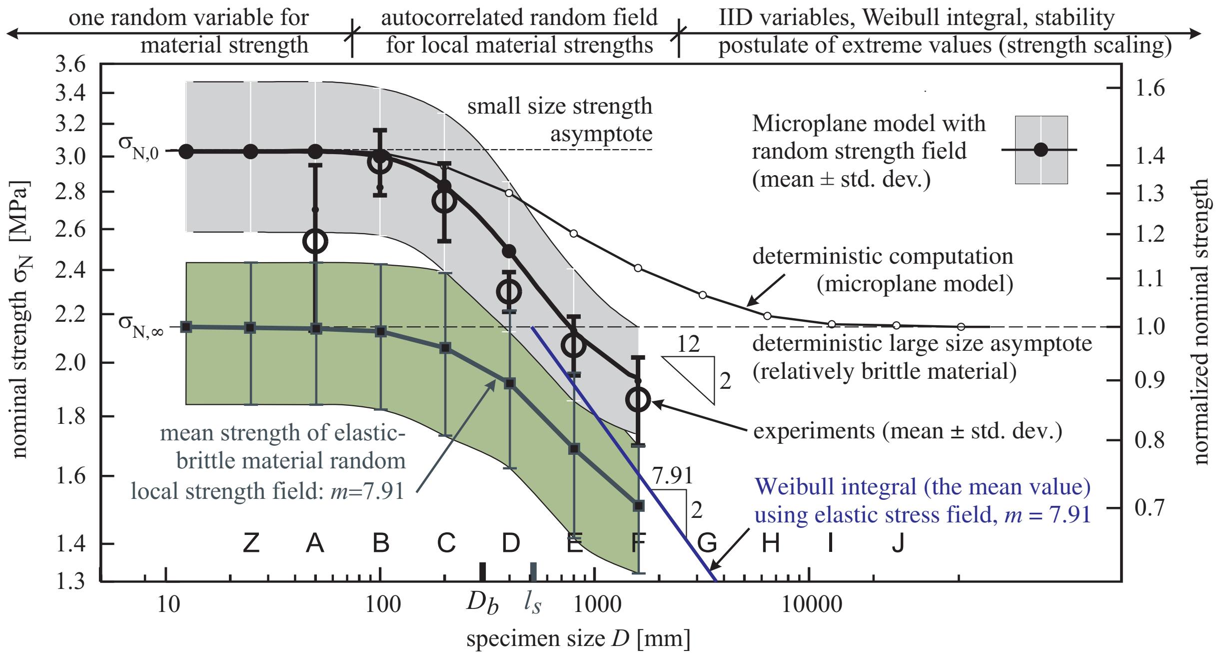

Figure 4: Comparison of results in a size effect plot.

5. Conclusions

We present a combination of non-linear computational mechanics tools with a simulation of random fields of spatially correlated material properties as an approach to the modeling of failure in quasibrittle materials. The performed numerical simulations of the random responses of tensile tests with dog-bone shaped specimens are in good agreement with the published data. We show that experimentally obtained data can be captured by the combination of all these size effects (deterministic and stochastic size effect, "weak boundary" effect and out of plane rotation of stiffness plate).

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